Bass Reflex Performance Envelope

The performance envelope that can be expected of a ported or passive radiator loudspeaker is outlined with a few simple graphs and easy to use models.

Performance Envelope Means:

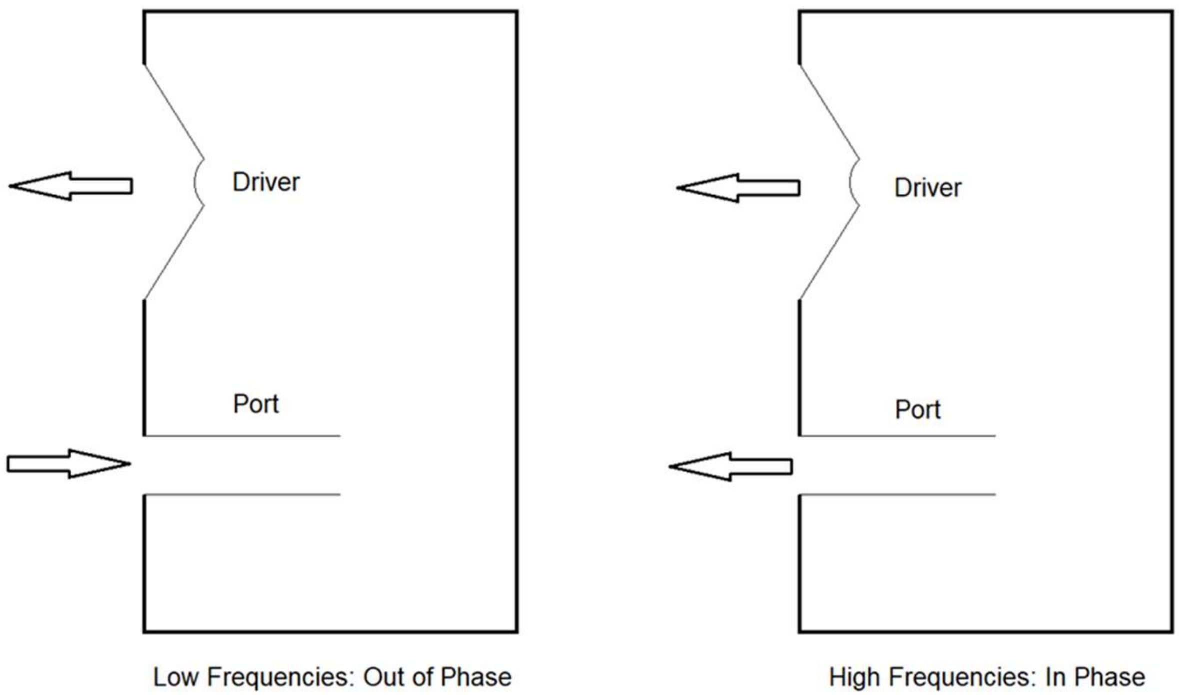
- How Loud?
- How Low?
- How Big?
- How much Power?

Getting there is our story today…

Gary Gesellchen
Feb 2020

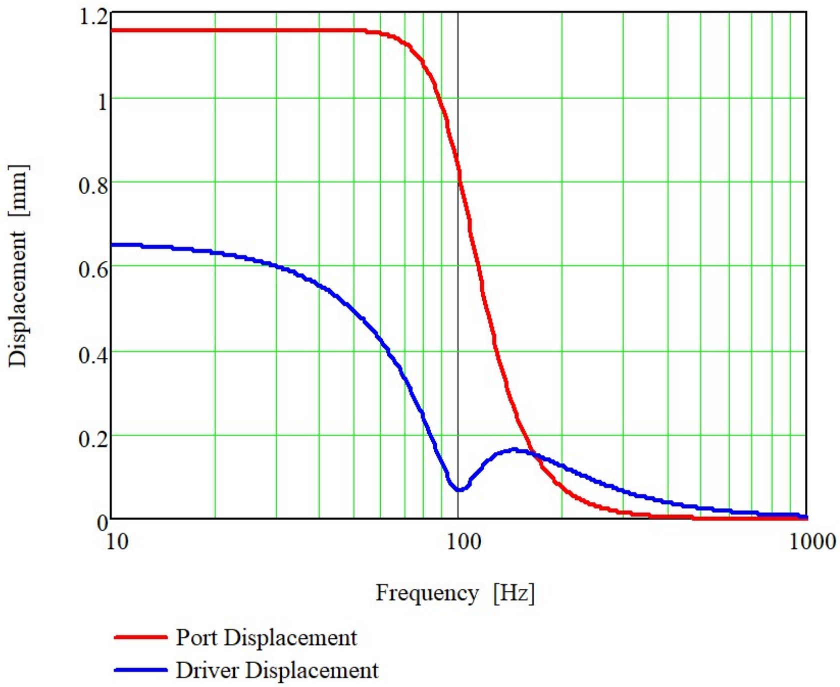
Vanatoo
Technology Serving Art

What is Bass Reflex?

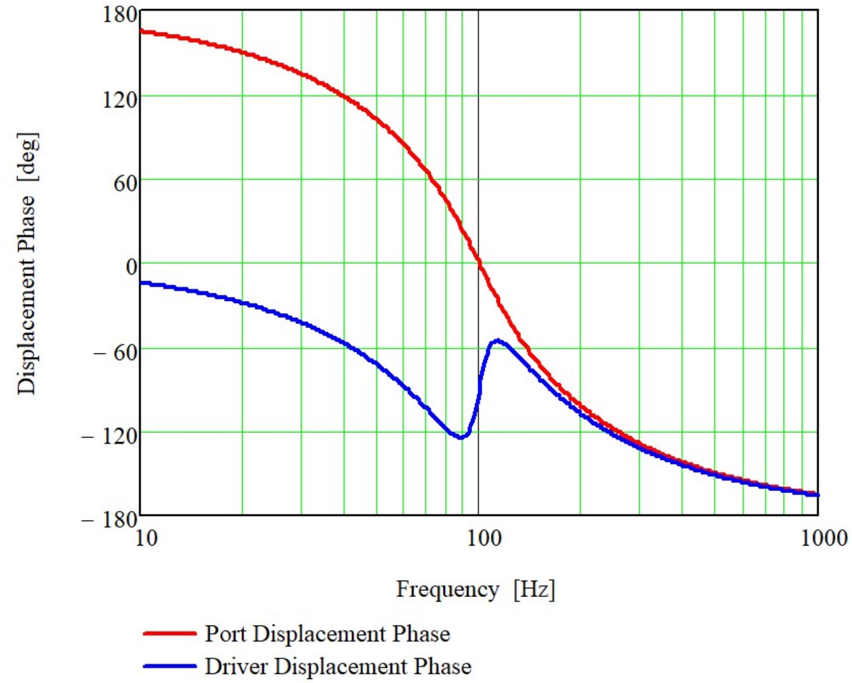


How Bass Reflex Works

Displacements vs Frequency



Displacement Phases vs Frequency



How much Sound is Generated?

(SPL of a Piston on an Infinite Baffle)

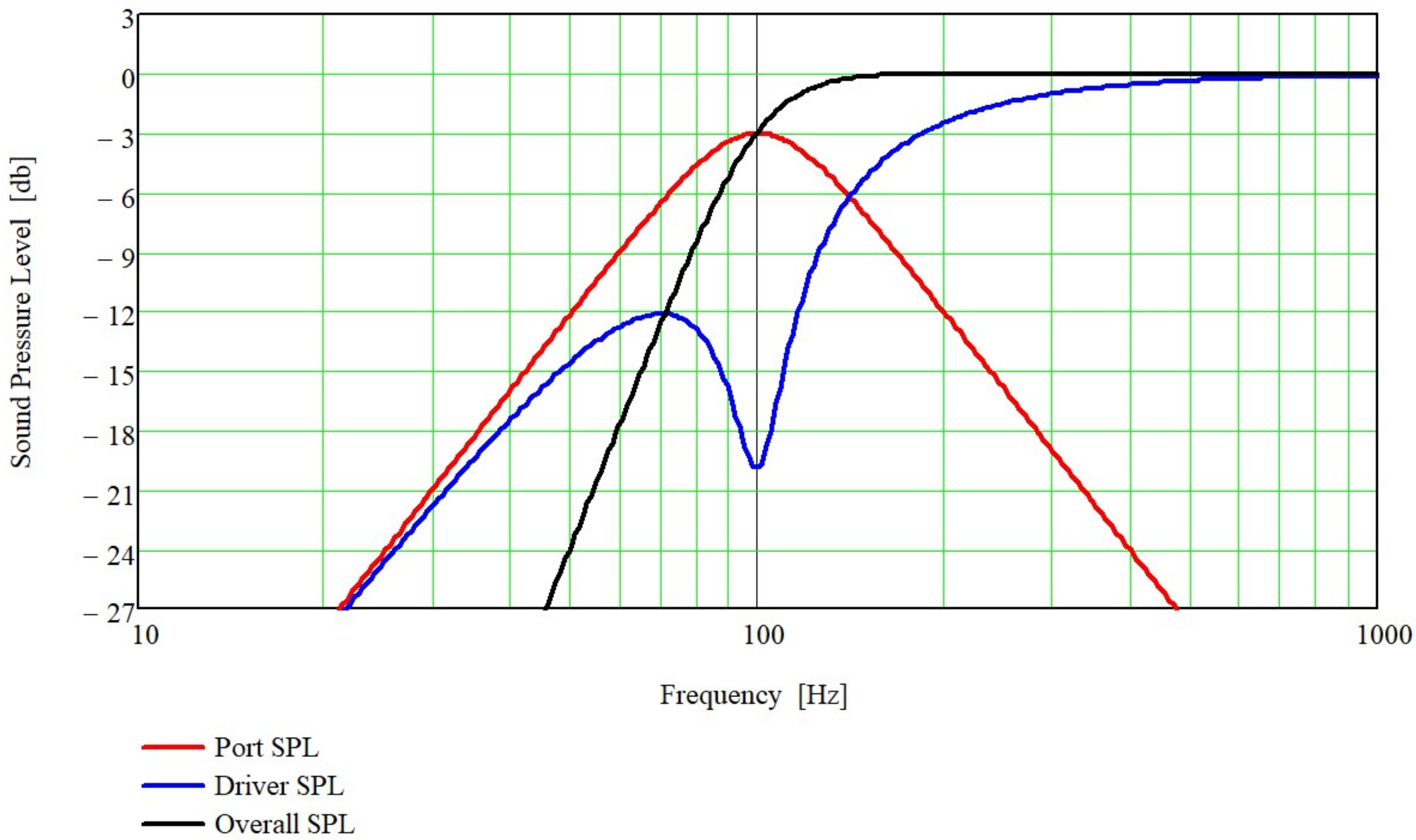
Displacement: $$SPL(f,S,X) = 20 * log\left(\frac{\rho * S * 2 * \pi * f^2 * X}{2 * 10^{-5}}\right)$$

Velocity: $$SPL(f,S,V) = 20 * log\left(\frac{\rho * S * f * V}{2 * 10^{-5}}\right)$$

Acceleration: $$SPL(f,S,A) = 20 * log\left(\frac{\rho * S * A}{2 * \pi * 2 * 10^{-5}}\right)$$

Where: f is the frequency [Hz]
ρ is the air density of 1.204 [kg/m^3]
S is the piston area [m^2]
X is the piston displacement [m/s]
V is the piston velocity [m/s]
A is the piston acceleration [m^2/s]

Sound Pressure Generated by Port and Driver



The Vanatoo Thiele/Small Model

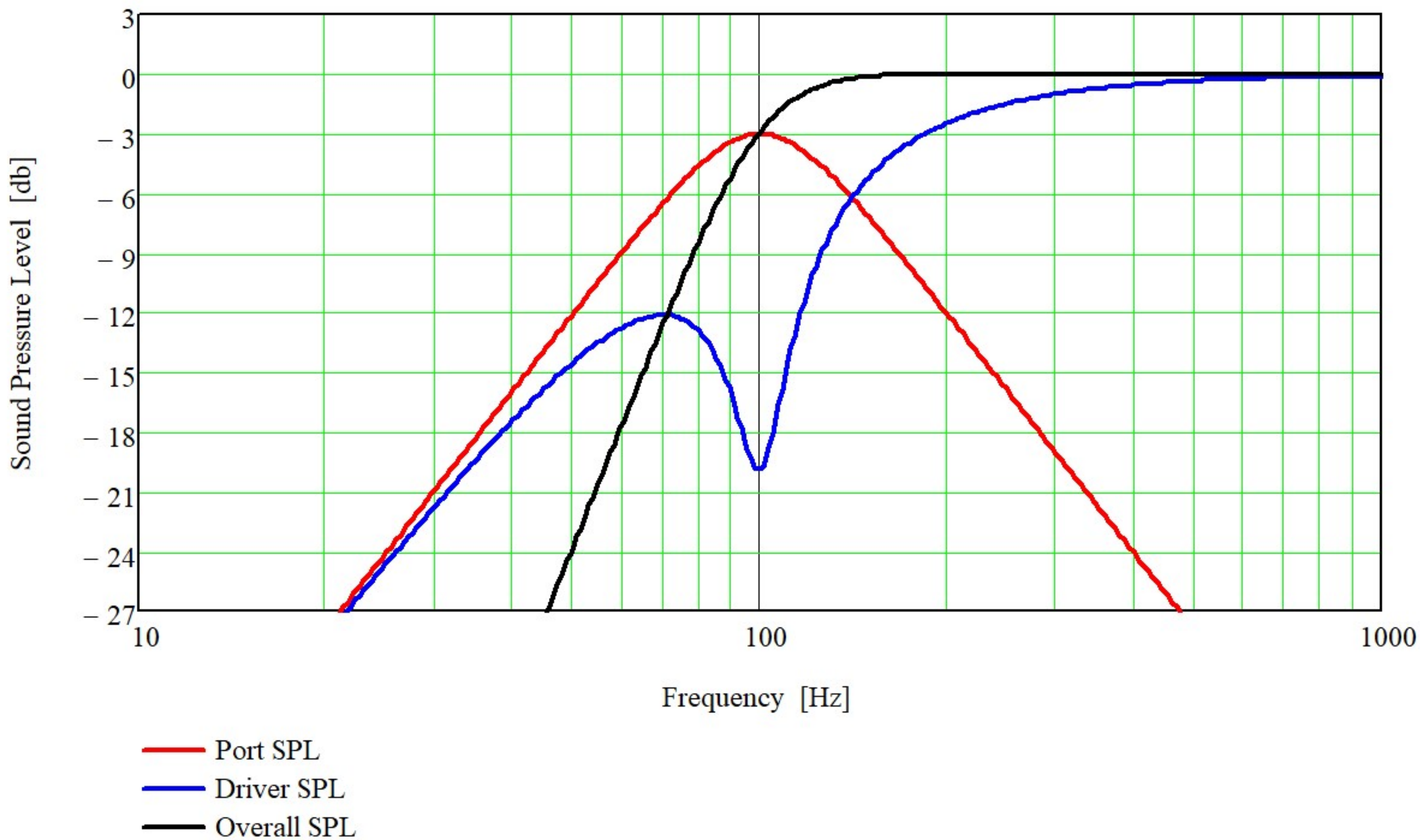
- Thiele (1961), Small (1970's) standardized the mathematical model.
- The T/S Model can be used to design to target response shapes, or "alignments".
- Gesellchen (2010's) put it in his language!
  - All model elements have the same real-world units as their respective physical elements.
  - Easily configured for mathematical optimization.

- T/S is a "Small Signal" Model:
  - Drivers are very nonlinear
    - Displacement, Current , Temp,…
  - Model not valid for large excursions, powers
    - Stiffness drops by 50%
    - Causes ~3dB error
  - My CEA 2010 testing usually shows more output
  - Going to use it!
    - Better than nothing

4^{th} Order Butterworth High-Pass

- Infinite possibilities
- 4^{Th} Order Butterworth (B4) is the best of all!
  - Lowest f3's with no "ripple"
  - Decent transient response
  - Reasonable box sizes
  - Reasonable power requirements
  - Lots of drivers designed around its requirements
  - Has really handy mathematical properties!

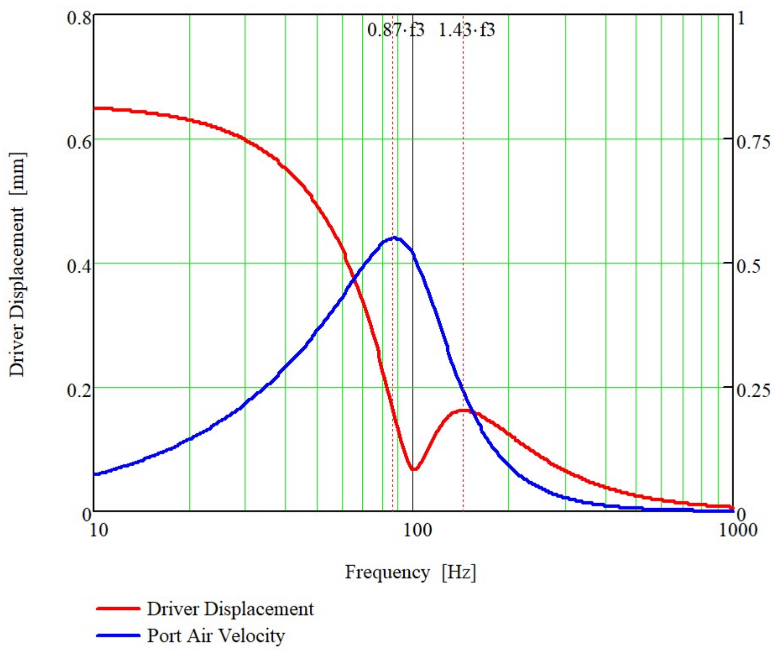
4^{th} Order Butterworth High Pass (B4) Response

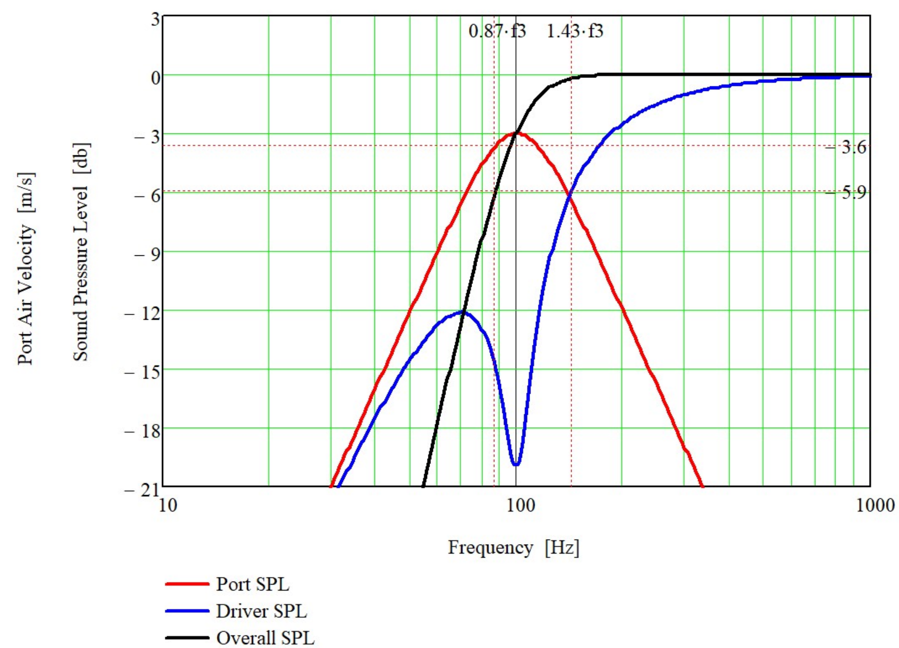


How Loud will it Play?

- The physical limits of the driver and port put a limit on the maximum SPL.
  - Driver is limited by the maximum controlled excursion, Xmax.
  - Port is limited by maximum noise-free air velocity, which we are assuming is 20 [m/s].
- These are frequency dependent.
- The B4 design will let us find SPL vs f3!

Insights: B4 makes the math *much* simpler!





- Port will always hit max air velocity at 0.87*f3
- Driver will always hit maximum in-band displacement at 1.43*f3.

- Port will always be 3.6dB down at 0.87*f3
- Driver will always be 5.9 dB down at 1.43*f3.

Port Output

- Velocity based equation for SPL from a piston.
- Maximum velocity is 20 [m/s].
- Mid-band SPL is 3.6dB higher than the SPL at 0.87*f3 (where the velocity peaks).

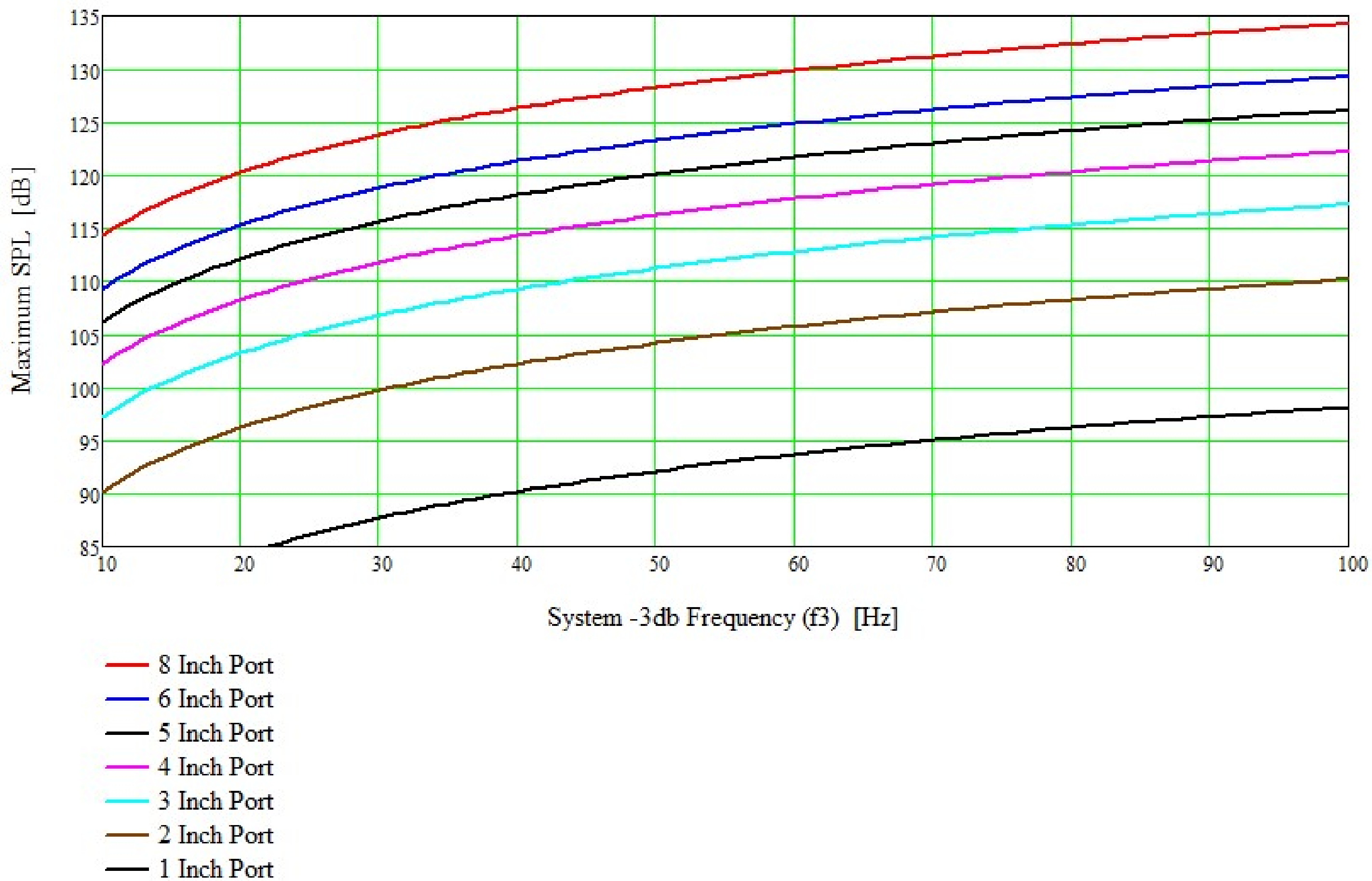
$$SPLM(f3, Dp) = 58.14 + 20 * \log(f3 * Dp^2)$$

Where: SPLM is the system mid-band Max SPL in dB

f3 is the system -3dB frequency in Hz

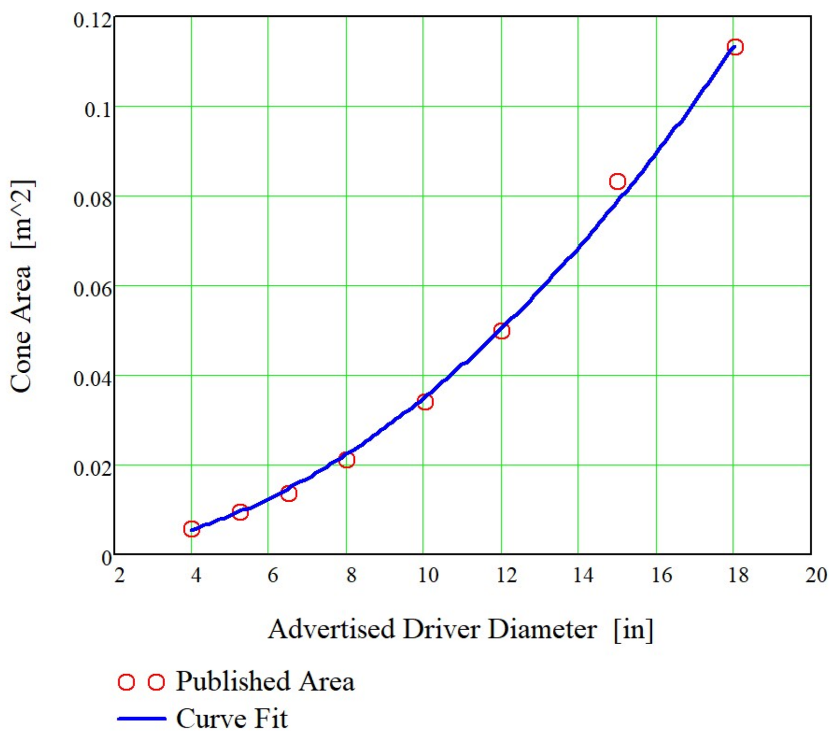
Dp is port diameter in inches

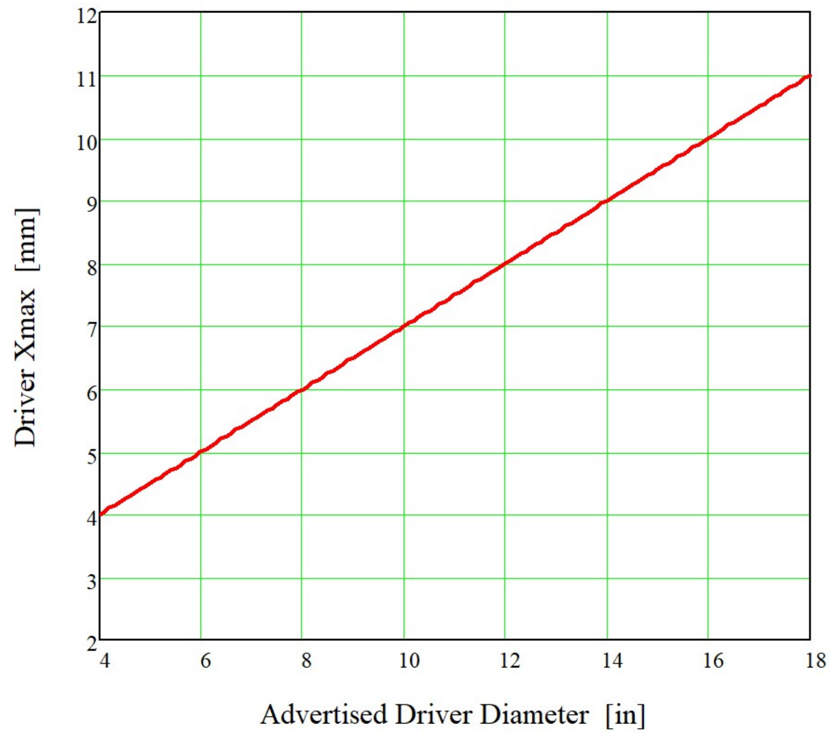
System SPL vs fs for various Port Sizes



The mid-band system Maximum SPL vs f3 for various diameter ports. If you want to play loud and low, *you need a big port!*

Driver Output: Commercial Driver Assumptions





$$Sd(d) = 0.00035 * d^2$$

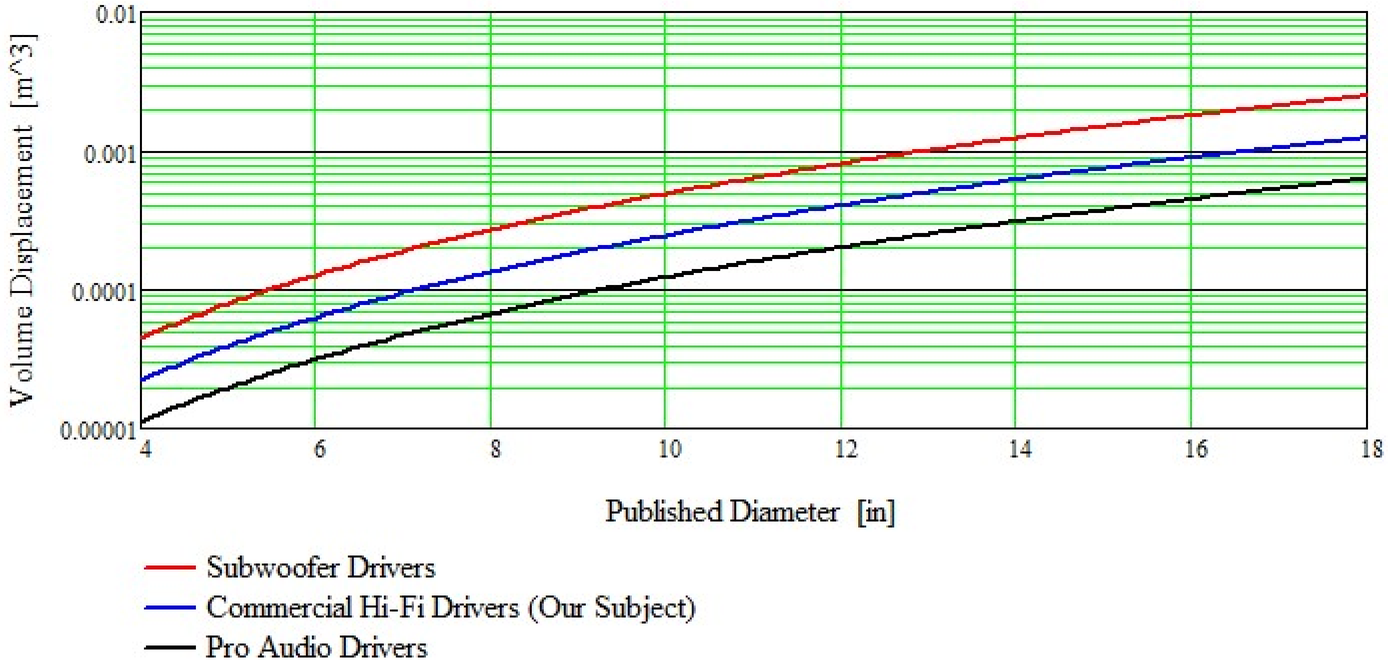
Where: d is published diameter [in]

Sd is cone area in [m^2]

$$Xmax(d) = 2 + d/2$$

Where: Xmax is the max driver displacement [mm]

Volume Displacement of Hi-Fi Drivers

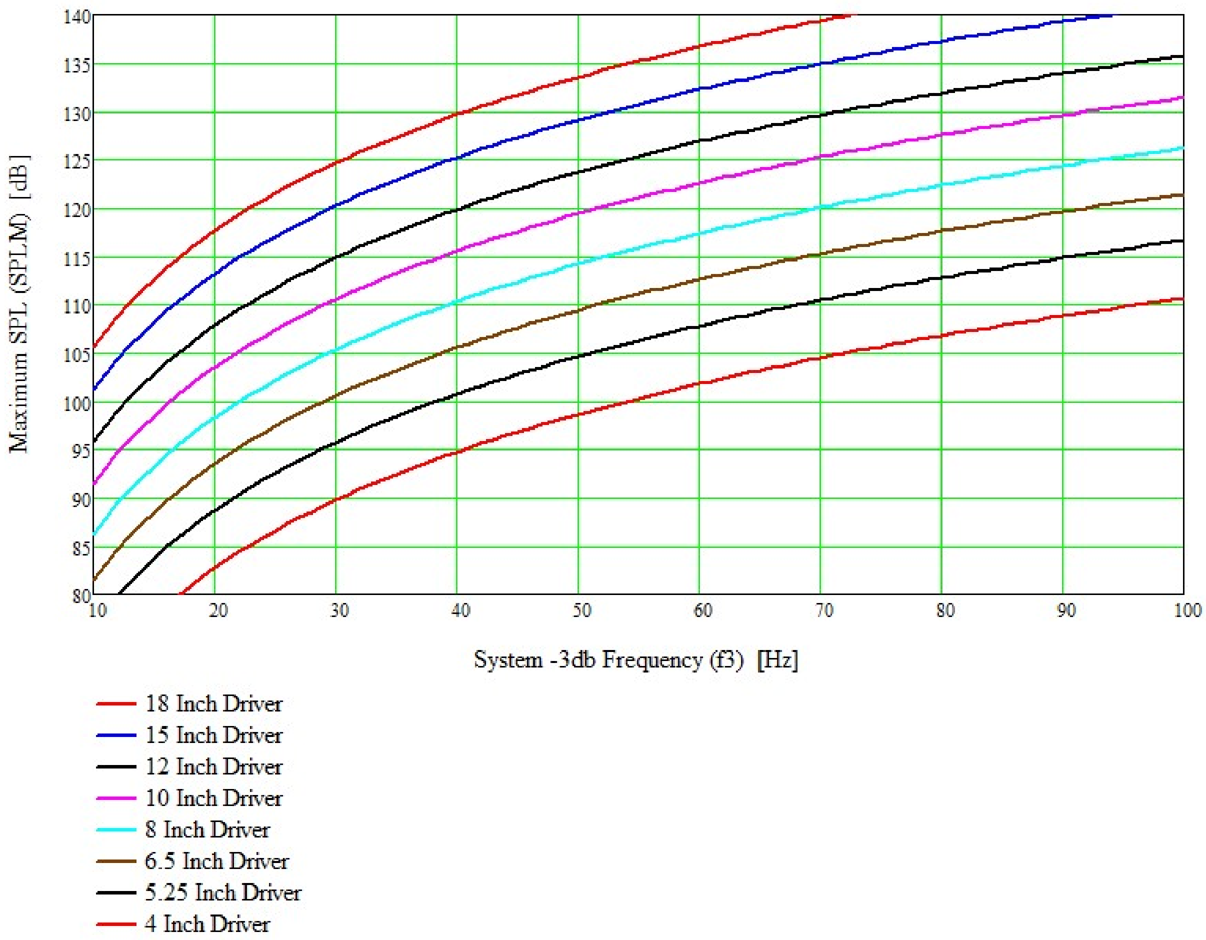


Leads to the Equation for Driver Output:

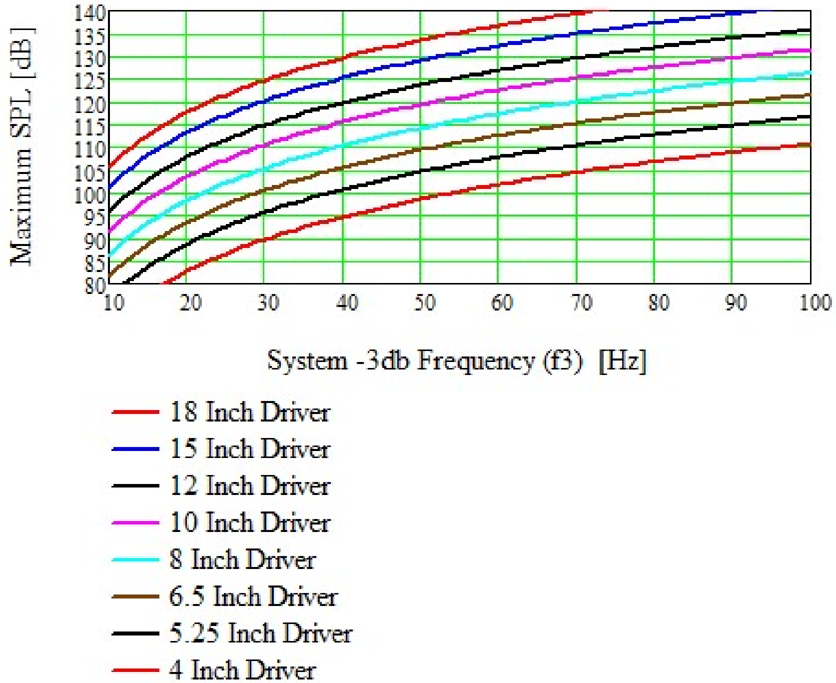
$$SPLM(f3, d) = -5.46 + 20 * \log\left(f3^2 * \left(\frac{d^3}{2} + 2 * d^2\right)\right)$$

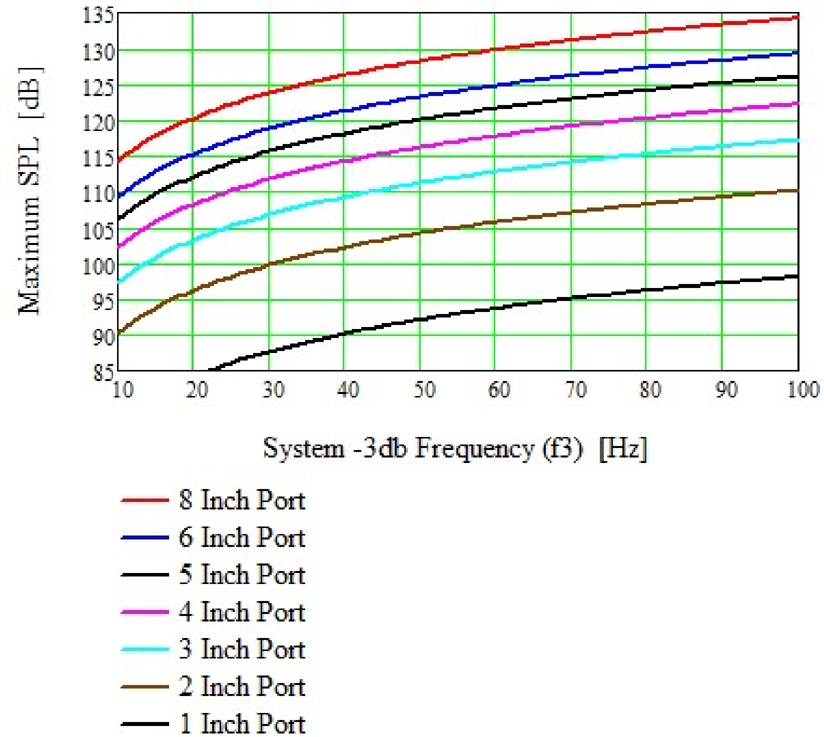
Where: SPLM is the system mid-band Max SPL [dB]
f3 is the system -3dB frequency [Hz]
d is the Driver Size [in]

System SPL vs f3 for various Driver Sizes



This is kind of remarkable!





If you know how loud and how low you want to play, these two graphs tell you how big of driver and port you'll need!

This is our First Half

- Designed around B4 response
- Reasonable assumptions about drivers
- Reasonable expectations of port

Simple graphs for driver and port sizes

What’s missing?

  – Box Size?
  – Input Power?

The Iron Law

LOUDSPEAKER DESIGN

Hofmann's Iron Law –

a curiously useful way of looking at the low frequency performance of loudspeakers...

HENRY KLOSS*

THERE HAS always been a certain willingness to suspend both dis-

One of the delightful things about the sealed-box, acoustic-suspension,

discuss its basic and completely dependable interrelationships: What hap-

30

AUDIO • MARCH 1971

Iron Law for B4

$$SPL(f3, Vb, Pe) = 55.62 + 20 * \sqrt{f3^3 * Vb * Pe}$$

Where: Vb is box volume [m^3]
Pe is electrical input power [w]

From which we get:

$$\mathrm{Pe}(f3, d, Vb) = 7.76 * 10^{-7} * \left(\frac{\left(\frac{d^3}{2}+2*d^2\right)^2 * f3}{Vb}\right)$$

Where: d is Driver Size [in]
Pe is electrical input power for SPLM [w]

This is our Second Half

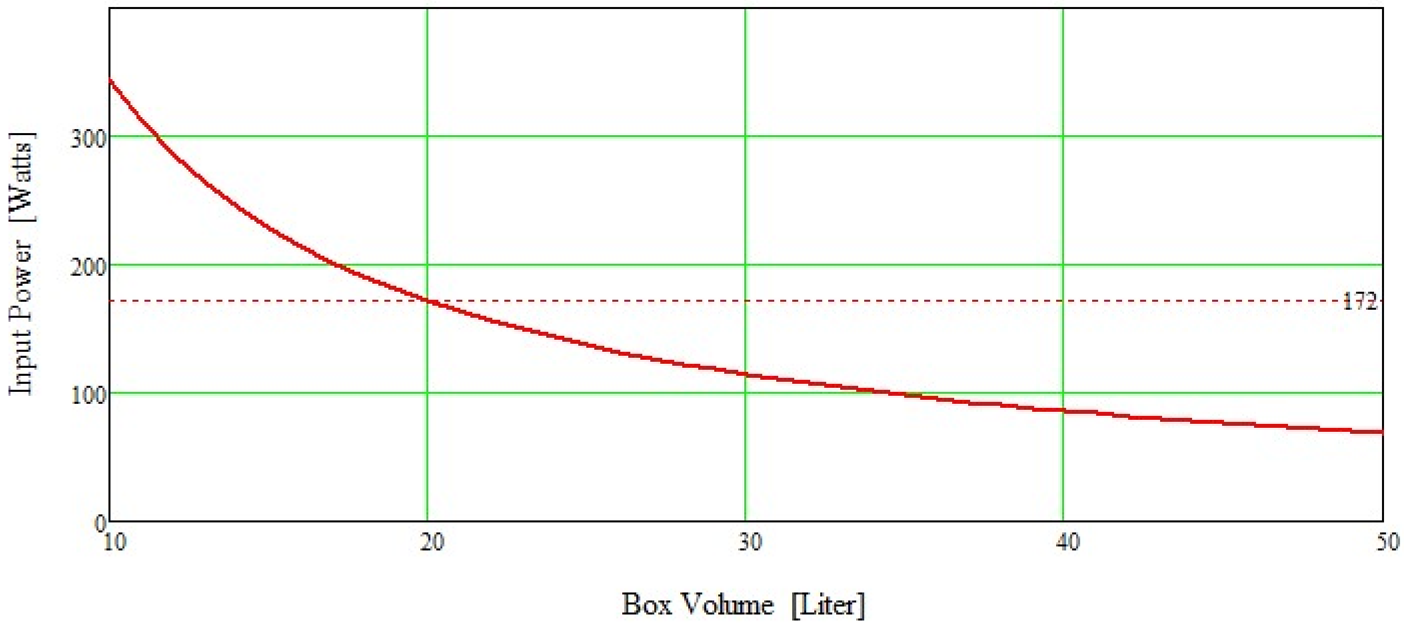
We know how the power will vary with the box size for any design we pick off of the graphs for SPL versus Driver and Port size.

There is no fixed design. Theoretically you can choose ANY size of box! Smaller boxes are going to require more power.

Design Example

- Credible Consumer Speaker
  - f3 = 30Hz
  - SPLM = 105dB
- Graphs say:
  - d = 8 inches
  - Port = 2.76 inches

Plot the Pe vs Vb Equation



Input Power versus Box Volume for a 8 inch driver with f3 = 30Hz and SPLM = 105.3dB.

20 liters is a largish bookshelf size with really good performance. 172 watts seems like a reasonable price to pay. *This is a nice design! It took 5 minutes!*

Only things left to figure out are the port length and the driver T/S requirements.

Port Length and Fit

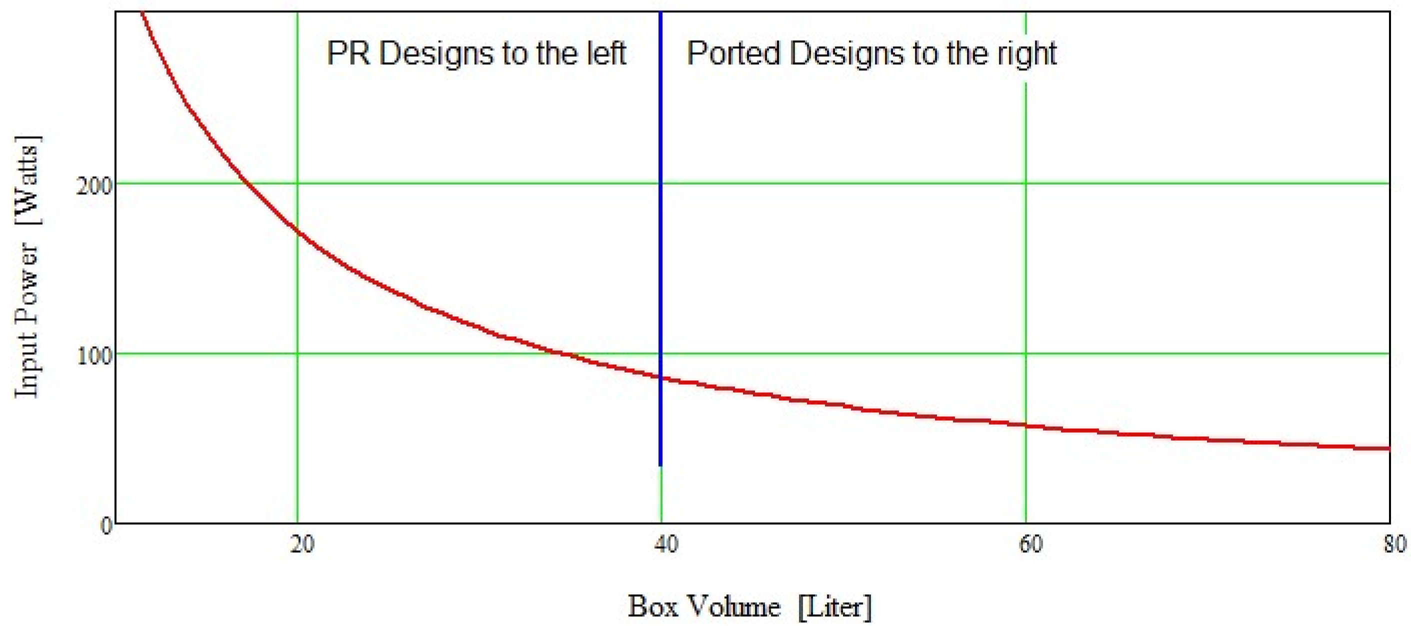
Box Tuning Equation:

$$Lp(f3, Vb, Dp) = 59.5 * \frac{Dp^2}{Vb * f3^2} - 0.73 * Dp$$

This gives 23 inches for our design example.

But, the port cannot be longer than ~ $\sqrt[3]{Vb}$ - Dp   or it won't fit in the box!

We can add this to our Power vs Vb graph:



Blue line is the smallest box that will fit the required port diameter:

- Passive radiator designs are to the left of the blue line.
- Ported designs are to the right of the blue line.

Our nice design really needs to be about 40 liters to fit the port and give the performance.

If we want a 20 liter box we'll need to either give up performance (f3 or SPL) or switch to a passive radiator.

Driver Parameters

- Every point on the SPL vs f3 curve for a given driver size requires different driver parameters.
- No guarantee that any particular combination is physically realizable.
- Spreadsheet is used to calculate the driver parameters.

What we've Covered

The really big design questions are:

- How Loud?
- How Low?
- Required Driver and Port?
- How Big?
- How much Power?

The designer gets to specify how loud and how low, which determines the driver and port size.

The designer gets to make the box size vs power decision.

Basic design outline can be done in a few minutes!

The Spreadsheets

- 100Hz B4 Model.xlsm

  Model showing a basic 100Hz B4 response.
- SPLM vs f3 and Power vs Vb for Ported B4 System.xlsm

  Model that allows the user to select the driver size and f3, SPL is calculated based on port, box size, and power. Driver T/S parameters are calculated and the system model is presented.
- General Purpose Ported T-S Loudspeaker Model.xlsx

  User can model any ported system.
- General Purpose PR T-S Loudspeaker Model.xlsm

  User can model any passive radiator system.