

Audio-Band Test and Measurement

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Audio Band Test and Measurement

Introduction

- 1. Brief overview of current audio related test and measurement hardware and software.**
 - I'm not going to get too specific on any one subject.
 - I'm not going to specifically pitch any one piece of software or hardware.
 - By 'current' I mean that I will be skipping some topics which are not **commonly** used anymore. For example, we will not be talking about measuring Transfer Functions with Stepped-Sine method.
- 2. How the software and hardware functions to perform the measurements.**
 - There are very few sources of information outside of the professional journals on how a audio measurement system is actually implemented.
 - Unlike most industries, test equipment and software aimed at the pro audio market tends to have poor documentation and a propensity to hide its functionality.
 - Understanding exactly how your test and measurement equipment works is a VERY important step in performing measurements you can trust.
- 3. Demo the proper setup and procedure for a variety of the measurements in the slides, with special attention to highlight limitations.**

Audio Band Test and Measurement

Introduction

This is not a introduction to ‘Digital Signals and Systems’, Physics, Loudspeakers, etc.

- There are many excellent books which cover the physics and application. (Beranek, Toole, Geddes, and etc.)
- I don’t want to talk or argue about the best way to mathematically describe ‘sampling’.
- We will skip subjects like: What is the FFT? And instead, talk about how to best use the FFT, and what are the implications of using the FFT.

I am going to be VERY light on the mathematics. I will try and use illustrations as much as possible.

- Obviously, some concepts are easier to understand when written out in mathematical terms, but there wont be any triple integrals or proofs of theorems.
- If you want to learn about the theorems and mathematics behind Digital Signal Processing (DSP) there are a number of OpenCourseWare video lecture series available online. (MIT, UT Austin)

Each chapter will introduce measurement concepts and we will discuss the theory and operation.

- Each section introduces some theory, not necessarily in order, which will be applicable in the sections that follow. For example, theory from chapter (1) will apply to chapter (4) just as much.
- I will include references at the end of each section. Especially when the state of the art is beyond my simplified explanation.

Please- STOP and ask questions !

- I wrote these slides in a very colloquial fashion. I would prefer to have more of a discussion than a one-way talk.
- We are going to cover a broad range of topics, we can stop and concentrate on something which you are interested in.
- If there is a particular piece of software or hardware you want to discuss, let me know.
- If anything seems to be in error please point it out. I am sure there are mistakes.

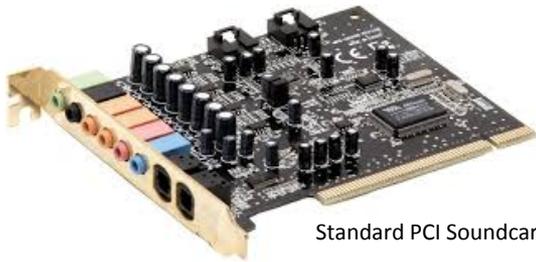
Chapter 1

Three Types of Measurement Hardware

- Single Channel
- Dual Channel
- Single/Multi Channel Coherent



National Instruments
PXI-4461



Standard PCI Soundcard



Audio Precision
SystemTWO



Firewire/USB Audio Interface



Audiomatica
CLIO 10

Chapter 1

Three Types of Measurement Hardware

Single Channel

- Handheld Sound Level Meters
- PC Sound Cards
- Cell Phones and Tablets

Dual Channel

- High-end audio interface

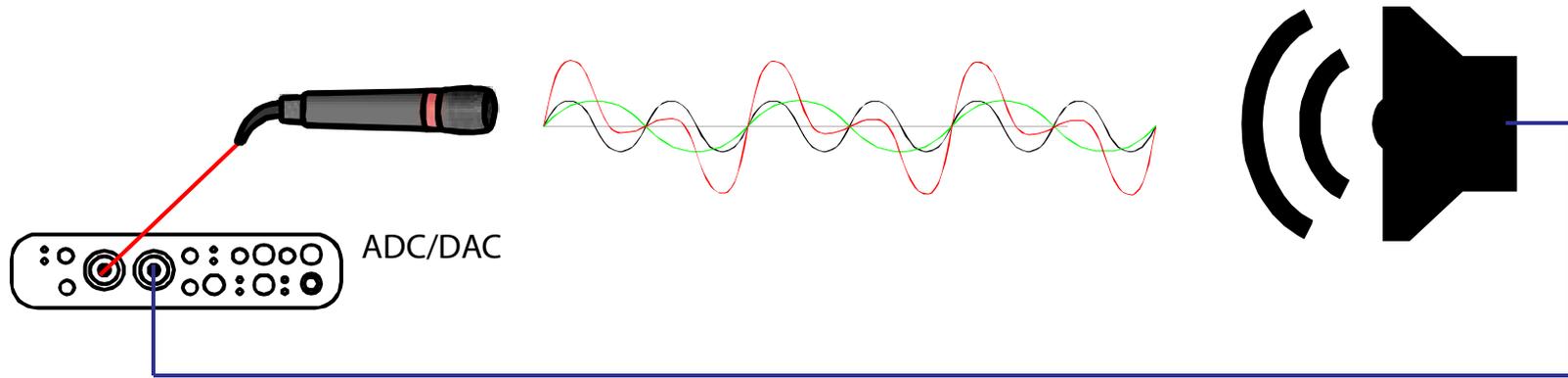
Single/Multi Channel Coherent

- High-end audio interface
- Dynamic Signal Analyzer
- Other Test Equipment designed for the audio industry

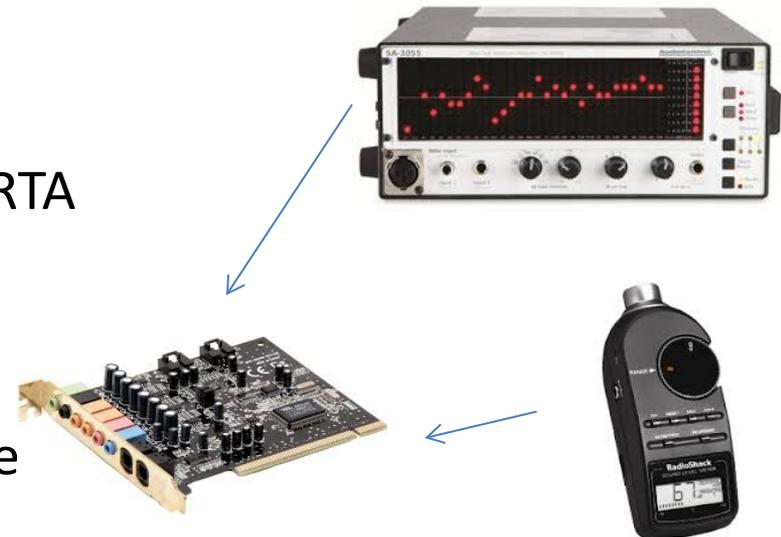
These are broad super-categories for discussion purposes only. They could also be the mode in which a device is used.

There are lots of commercial off the shelf (COTS) devices which fall in between categories.

Single Channel Measurements



- We generate a deterministic signal or random signal with known distribution.
- Sound Level Measurements
- Spectrum Analyzer - Power Spectrum / RTA
- Great for understanding whole room.
- Requires calibration of all parts of signal chain.
- Easy to setup and works on any PC, table or cell phone.



Single Channel Measurements

What is wrong here if we care about timing?

1. Clocks are not synchronized between ADCs/DACs

CLK1 != CLK2

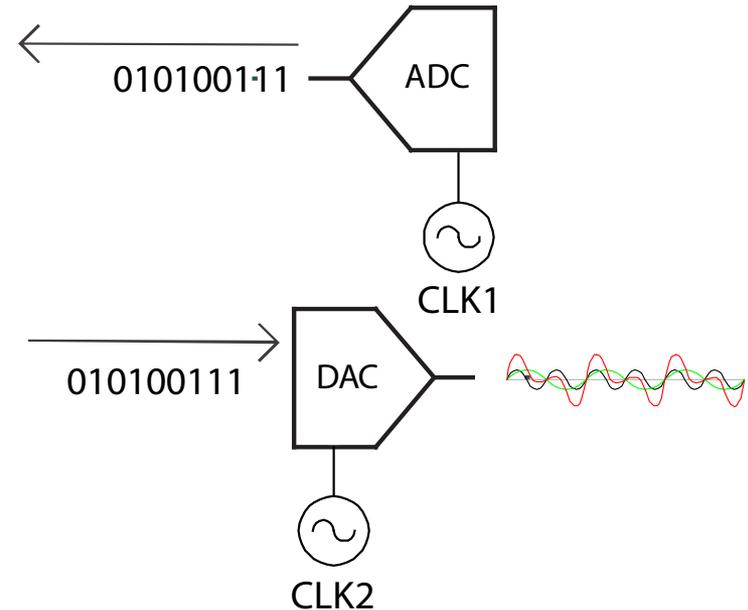
In some cases there may be a non-constant phase relationship between the ADC/DAC. Signal generator is not even the same device as the analyzer.

- Separate Signal Gen and Analyzer
- Some PC Sound cards

CLK2 = CLK1 + phase

Most often there is some phase difference between the clock seen by the ADC vs. the DAC.

- Some PC Sound Cards



Single Channel Measurements

What is wrong here if we care about timing?

2. When we push the start button we are at the mercy of the OS and DAQ drivers.

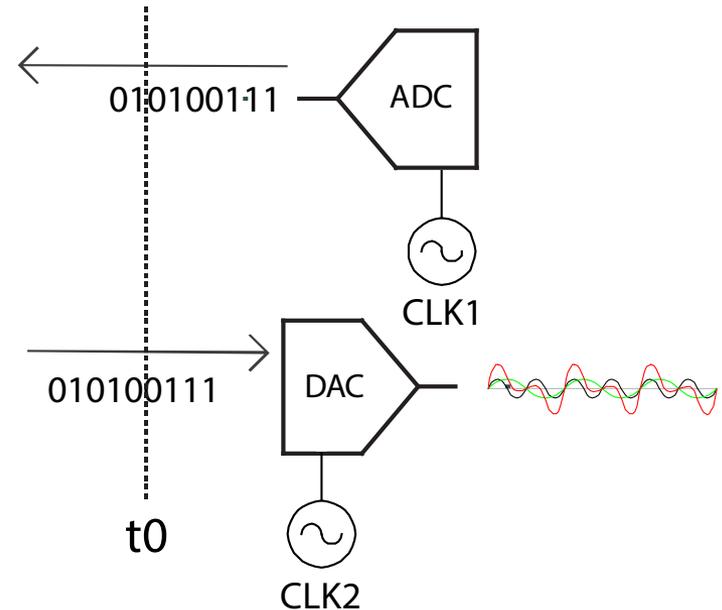
Software Triggering

How do we know where the first sample is?

- Software Starts Recording
- Software Plays the Stimulus Signal
- Other programs may execute at random between the two events

Calibrate the Delay out

- Even if we calibrate for the average delay there is still variability.
- Increasing the sample rate will reduce the effect in practice.



Single Channel Measurements

Questions / Additional Items

WDM/ ASIO/Mac/PC/Linux?

- We will touch on this in the next section.

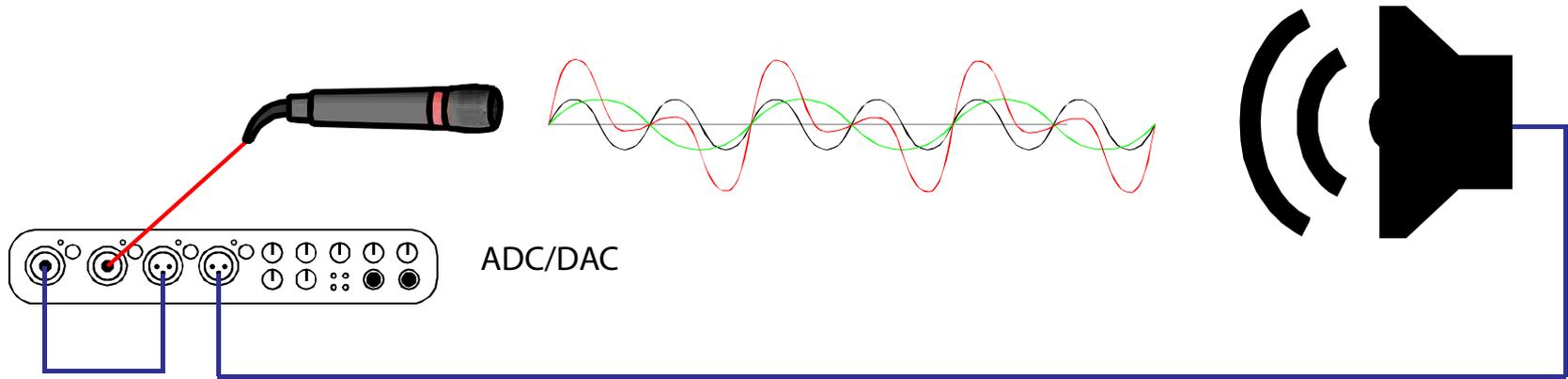
Special Purpose DAQ Devices?

- Example EASERA Firebox, CLIO, NI, etc.
- We will talk about these in the last section.

Why should I even care? My iPhone RTA works better/easier than SMAART!

1. Phase information is important if you want to setup your loudspeakers / crossovers properly.
 - If your timing is off, variations in the phase can become buried in the delay.
 - Most good quasi-anechoic measurements depend on accurate timing to calculate an accurate magnitude response. We will talk about this in the next chapter.
2. If you are building loudspeakers/enclosures/crossovers you are **crazy** not to have a proper measurement system.
 - You should be measuring your loudspeaker, not the loudspeaker + room. Your measurements should be reproducible by anyone in any room.
 - Building a loudspeaker that works well in a variety of situations should be the goal.

Dual Channel Measurement



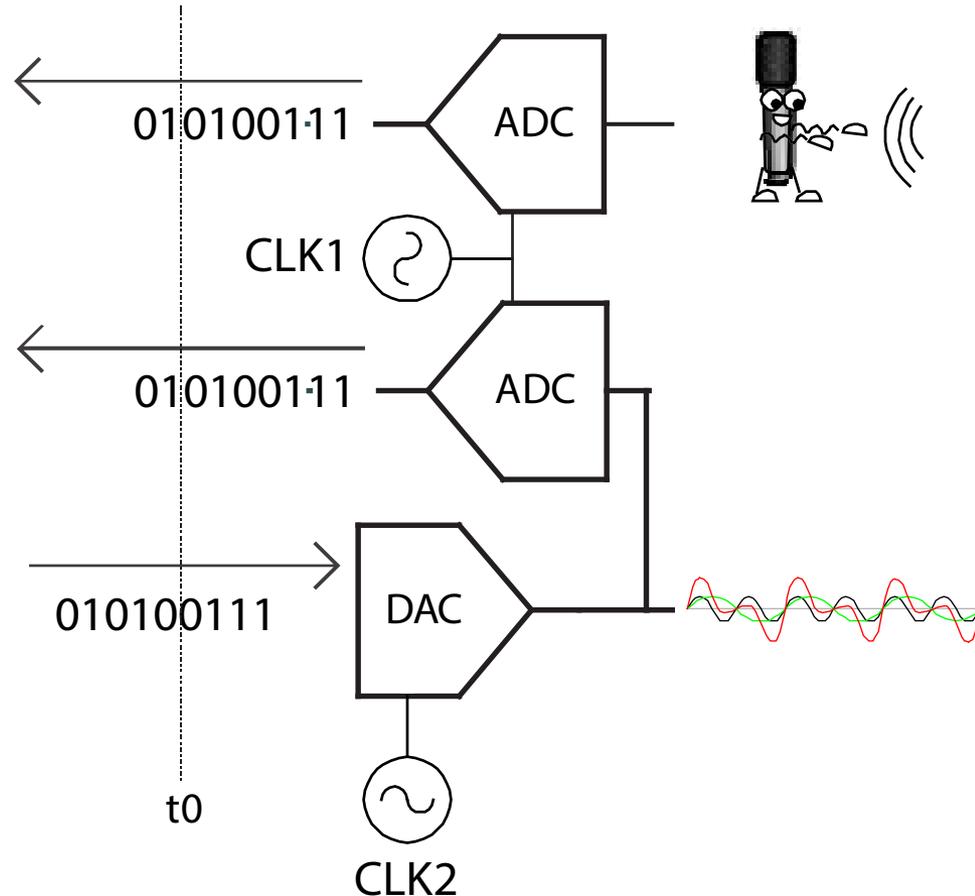
- Our stimulus can be any signal which excites the bandwidth we are interested in.
- Sound Level Measurement / Spectrum Analyzer / RTA
- Transfer Function Measurements (Magnitude/Phase)
- Only Requires Calibration of Microphone and Amplifier
- Easy to setup, Works on any PC, much harder to implement on Cellphone or Tablet
- Fixes most of the issues with the single channel measurement.

Dual Channel Measurement

Improvements over the Single Channel Measurement

We use two ADCs with a coherent clock.

- Since most consumer ADC/Codecs are stereo pairs, this is very easy to implement even on the cheapest hardware.
- If Our Measurement system utilizes more than just a single stereo ADC, careful attention must be paid to the clock distribution.
- Every year the drivers provided by high end audio interface manufactures get better.



Dual Channel Measurement

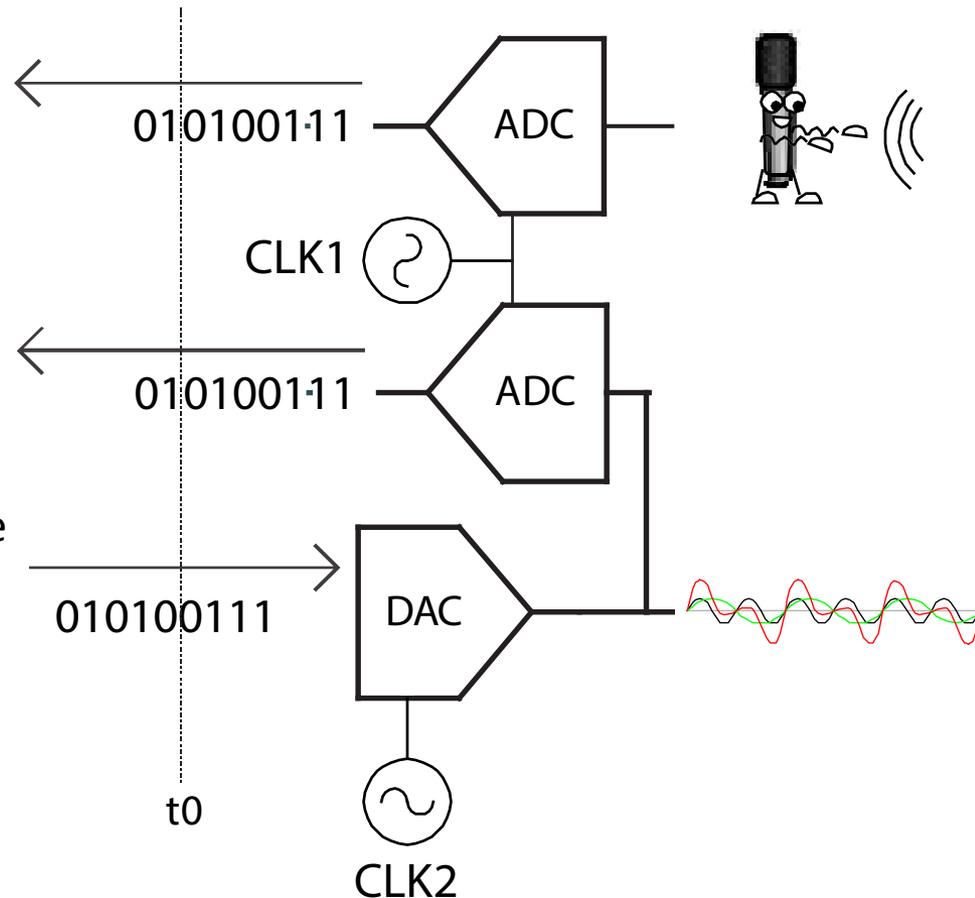
Improvements over the Single Channel Measurement

No need to calibrate the entire signal path.

- We can take our loopback reference signal from as close to the amplifier as possible. We only need to compensate for the microphone and amplifier output.

Stimulus doesn't need to be deterministic or random with known distribution.

- There is no need for the stimulus to come from the measurement system.
- It is practical to use any signal, including live music, to determine the Transfer function.



Dual Channel Measurement

Issues with Dual Channel Measurements

One ADC is used as a reference and can not be used for measurements.

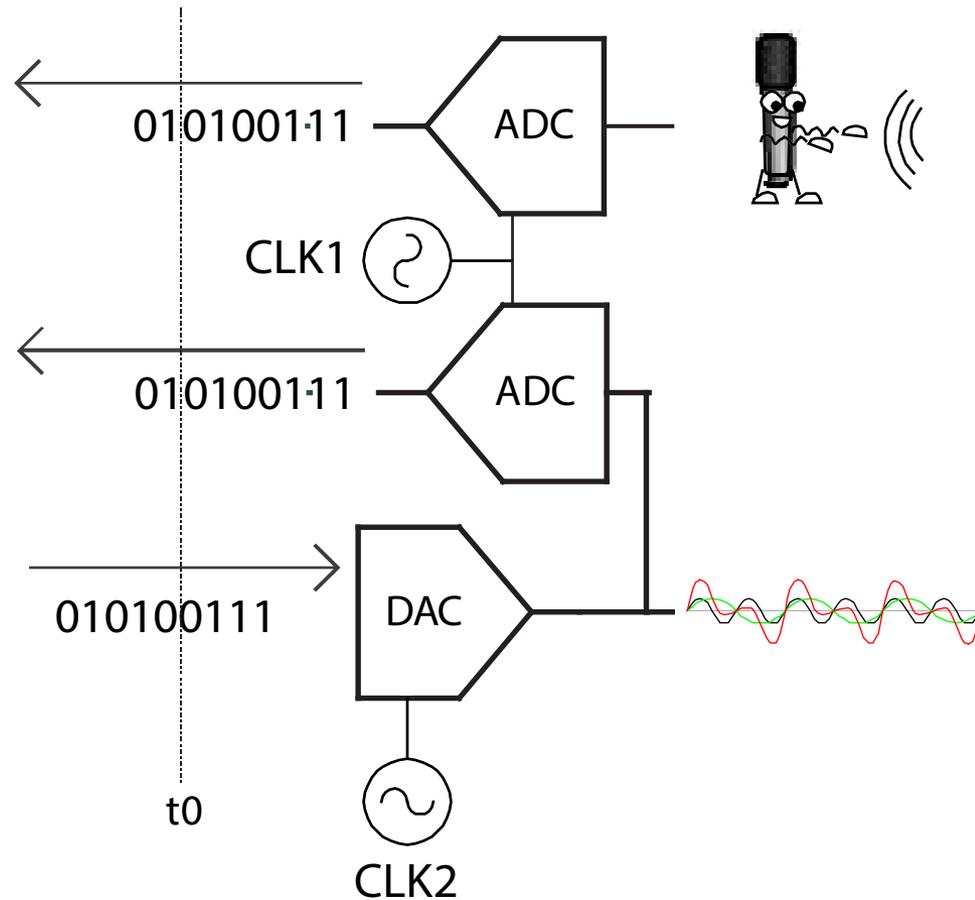
- In a portable USB powered solution this can be a deal breaker.

Still no way to accurately start DAC/ADC at the same time.

- Even if the clocks are coherent software triggering introduces jitter into the relative position of data stream.

No way to ensure the output data clocks are in sync if you have multiple ADCs.

- Different power on times.
- No SYNC pin (next section)



Dual Channel Measurement

Questions / Additional Items

ASIO Drivers and high end audio interfaces

- Although there is not any hardware triggering, GOOD ASIO Drivers significantly reduce the variance in delay between starting playback and recording.
- Assuming the host PC is in good condition and has the processing power and bandwidth to capture the data.
- A high end audio interface running at 96kHz with manufactures ASIO driver will have negligible variance for 99% of applications.
- Applications like SMAART, REW, Sys-Tune, and etc. make estimations of delay which are invariant to this. (I think?)

Calibration

- If you are doing anything besides relative comparisons, your measurement hardware is only as good as its calibration
- eBay is full of 'okay' Microphone calibrators. On occasion you can get great deals on Brüel & Kjær 4231, less than \$500.
- Build a set of cables so that you can measure each part of your signal path.
- Get a good NIST calibrated multi-meter and signal generator. The HP 34401A 6.5 digit meter can be found for cheap. Older Wavetek signal generators are a good buy also, just make sure you get one which goes low enough in frequency.

Noise Problems

- Noise coupling into your measurement system is a really common problem. Good transformer isolated single-ended to differential conversions are a must.
- You should measure and compensate for if necessary any isolation transformer.

Single/Multi Channel Coherent Measurement



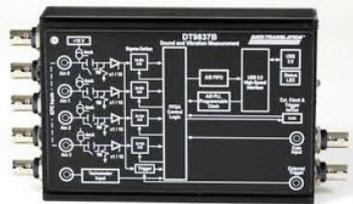
Audio Precision
Rackmount + more



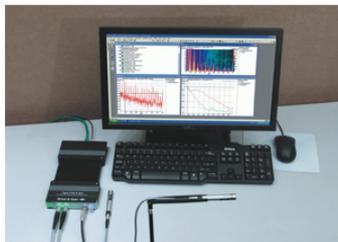
Agilent/Keysight
Standalone/Rackmount



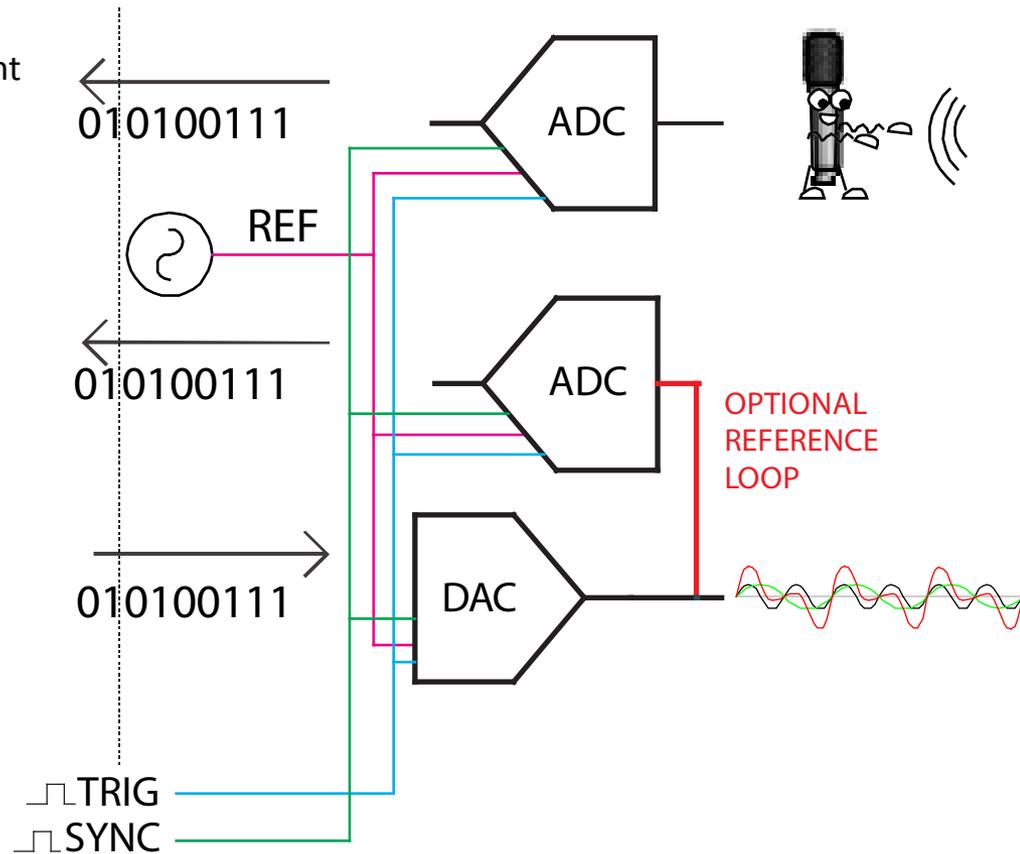
National Instruments
PCIe/PXIe/USB



Data Translation
USB



Brüel & Kjær
Rackmount + more



Single/Multi Channel Coherent Measurement

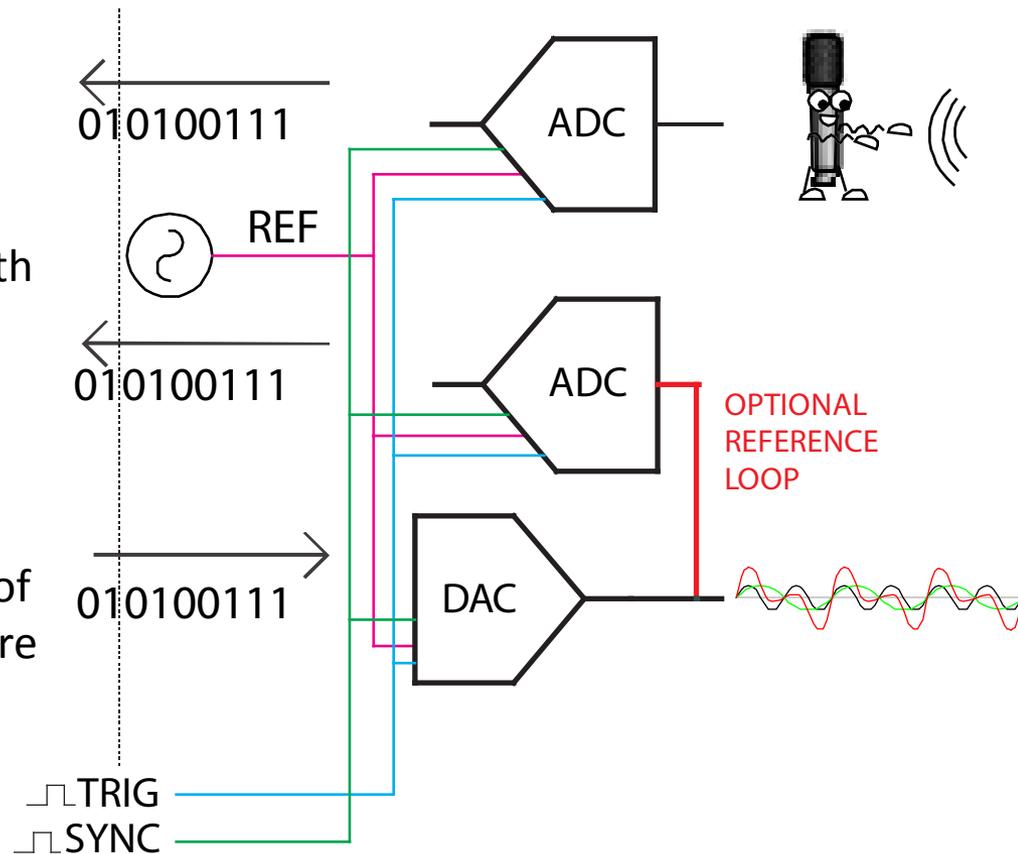
Improvements over Previous Examples

Coherent Reference Clock shared by ADC/DAC

- Manufacturers spend lots of time ensuring good PCB design.
- High stability oscillator with equal length path to all ADC/DAC devices.

Digital SYNC input to synchronize ADC output clock and DAC input clock.

- For multi-channel devices, at the start of each measurement all ADCs and DAC are held in this state to realign out/in data clocks to reference clock.



Single/Multi Channel Coherent Measurement

Improvements over Previous Examples

Digital TRIG input to accurately start measurement.

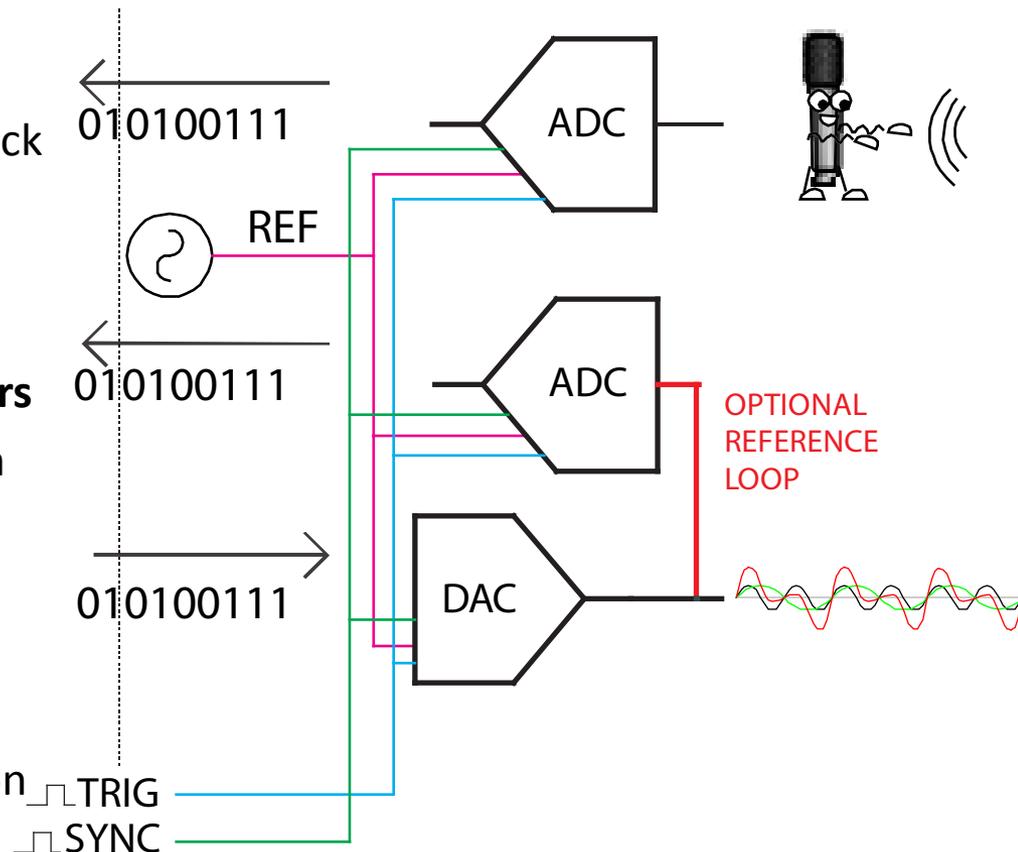
- A hardware trigger ensures that playback and recording start exactly at the same instant aligned to the REF clock.

Large on-board FIFO buffer / DMA transfers

- Allows continuous flow of data to/from host PC with no missing samples.

We can still use loopback channel from previous example system.

- To simplify calibration there is no reason we can't do a dual-channel transfer function measurement.



Single/Multi Channel Coherent Measurement

Questions / Additional Items

You said in the last section a dual channel setup with good ASIO drivers is good enough, why would you want this expensive stuff?

- Most likely you won't want to spend \$10,000 on your measurement system. The high end equipment is typically stationary and special purpose.
- If your primary use for measurements is walking into a venue or install job, setting up and EQing a system, a general purpose dual channel audio interface is the best equipment for the job.

That being said

- Occasionally you can find higher end equipment on eBay for very cheap.
- The USB based modules from NI and Data Translation are priced competitively with pro audio interfaces, they just won't work as a sound card device in Windows.
- If you are interested in Loudspeaker design as a hobby, or are starting a new loudspeaker manufacturing business, I would suggest making the investment.
- Being able to test and catalog incoming OEM drivers is VERY important. Failure rates as high as 1 in 3 can and do happen; don't let your customers discover the failures for you.

Any other questions before we move on to measurement software?

Chapter 2

Methods for Measurement

Introduction –

- Measurement Philosophy
- Fourier Transform and Sampling

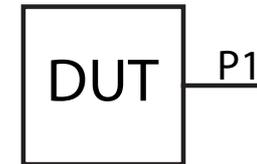
One Port

- Digital SPL Meter
- Spectrum Analyzer / RTA

Two Port – Transfer Function

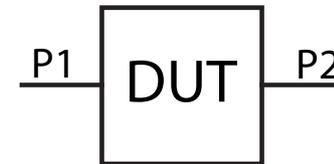
- Direct Form
- Heyser's Time Delay Spectrometry (TDS)
- Matched Filtering

Device Under Test (DUT)



One Port-Scalar Measurement

The DUT does something and we measure it.



Two Port-Vector Measurement

We input something to the DUT on P1 and measure what the DUT does to it on P2

Methods for Measurement

Introduction

Measurement Philosophy

We (as humans) strive to describe the physical world we live in by developing mathematical models.

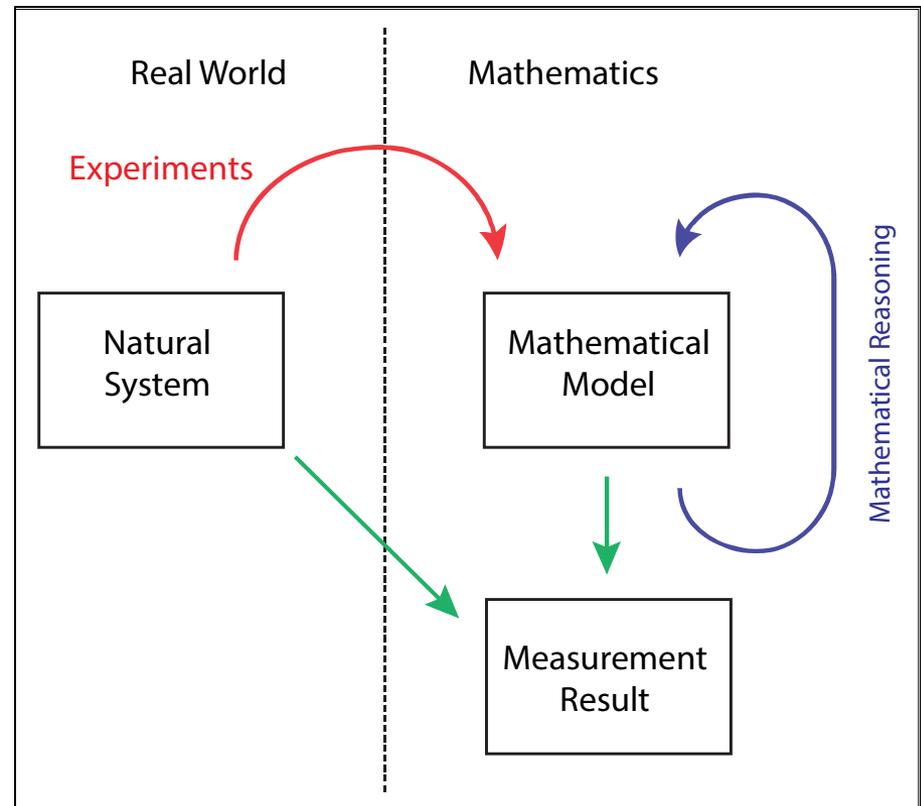
- Scientists perform experiments and develop mathematical models which fit the experimental data.
- Scientists make predictions from their mathematical models and perform experiments to verify the model is correct.
- Mathematicians and physicists, using mathematical reasoning extend the models, which are then empirically verified.

Models in some fields change quickly as the science evolves quickly, some models are very slow to change.

- Have you opened your High School chemistry book lately? Almost guaranteed it is utterly outdated.
- Many models depend on the scale or other constraints, but may have been replaced by new models at a different scale or under different constraints.

When a model has significant scientific consensus we tend to think of it more as a law, theory or something handed down by God.

If models remain unchallenged for a period of many years, the debate between scientists might be comparable to a Holy-War of the middle ages.



Methods for Measurement

Introduction

Measurement Philosophy

When we perform measurements, we collect real world observations and apply the correct mathematical model to them. (I.E. the applied sciences.)

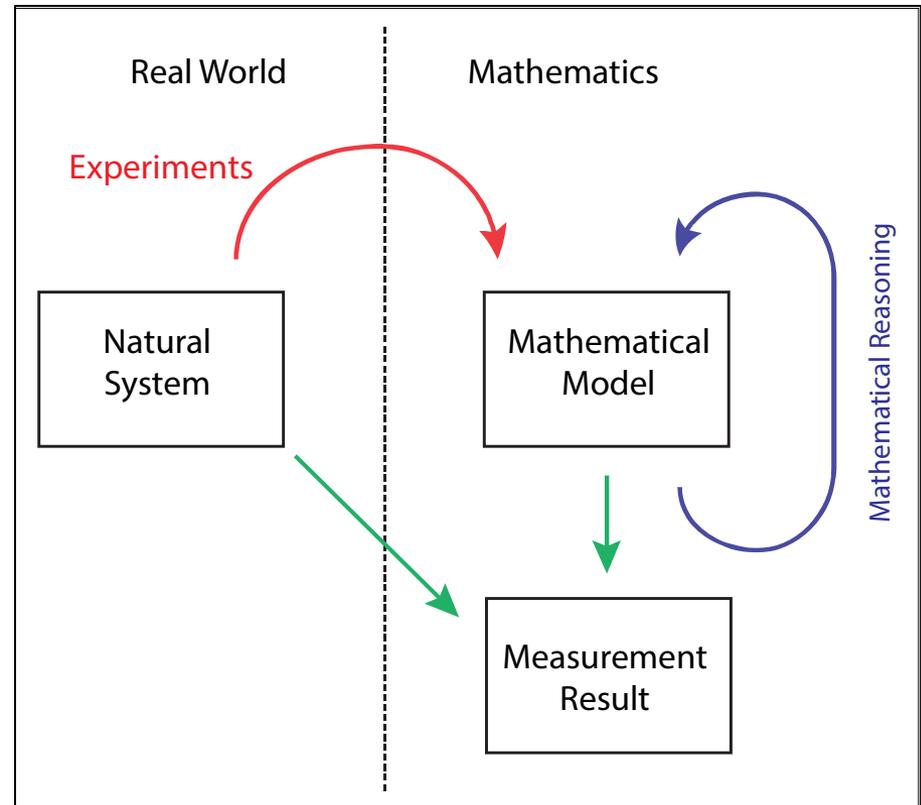
It is from this application of the model which we derive our measurement result.

Some of the models in our applied field which we are familiar with and commonly use:

- Fourier Transform
- Wave Equation
- Maxwell's Equations
- Ohm's Law
- Small Signal Model of a Speaker
- Sampling Theorem
- Linear Time Invariant (LTI) systems

The other 25-50 which I can't think of right now!

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Methods for Measurement

Introduction

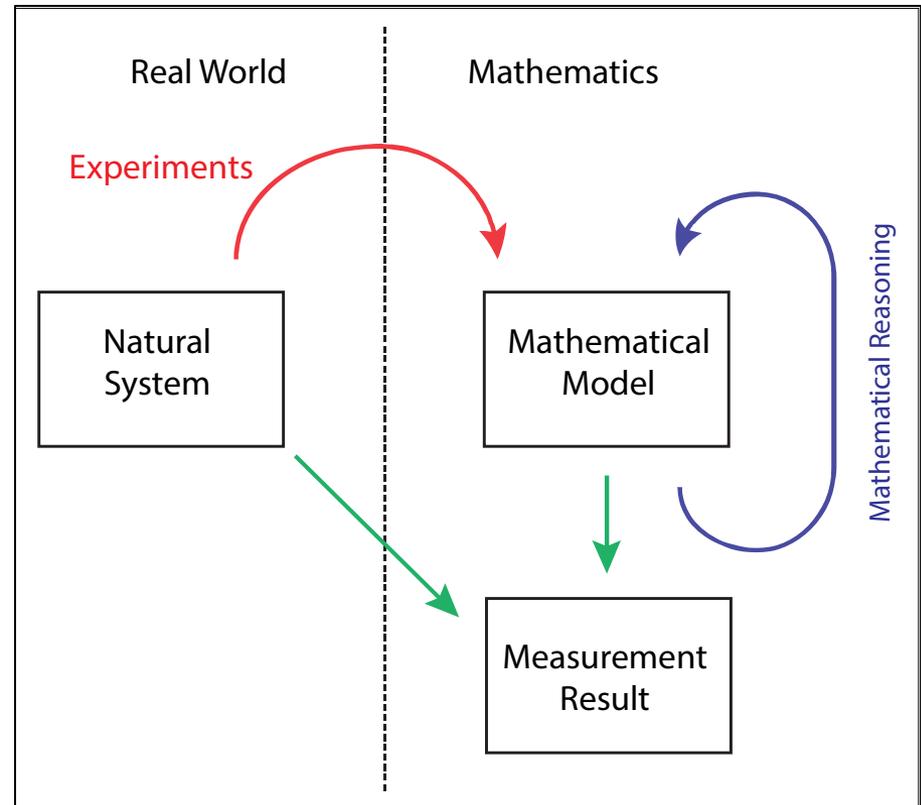
Measurement Philosophy

Of these, we will spend a lot of time discussing the Fourier Transform and LTI systems in this presentation.

Just a few things to keep in mind before we go on, that we will talk about in depth:

- Typically when we use the term, 'Frequency' we are talking about it as defined by the Fourier Transform, and not as it is defined by say a note on a sheet of music.
- We assume that the signals we are measuring can be neatly decomposed into sinusoids and that the composition makes sense.
- When we make most measurements of loudspeakers, we are making many assumptions that the system fits into the LTI model.
- To expand, we assume that we can select some chunk of the signal localized in time, and that it is a good example of the signal as a whole.

We should keep an open mind, that there are limitations to our current techniques and that much better techniques may be developed in the future.



Methods for Measurement

Introduction

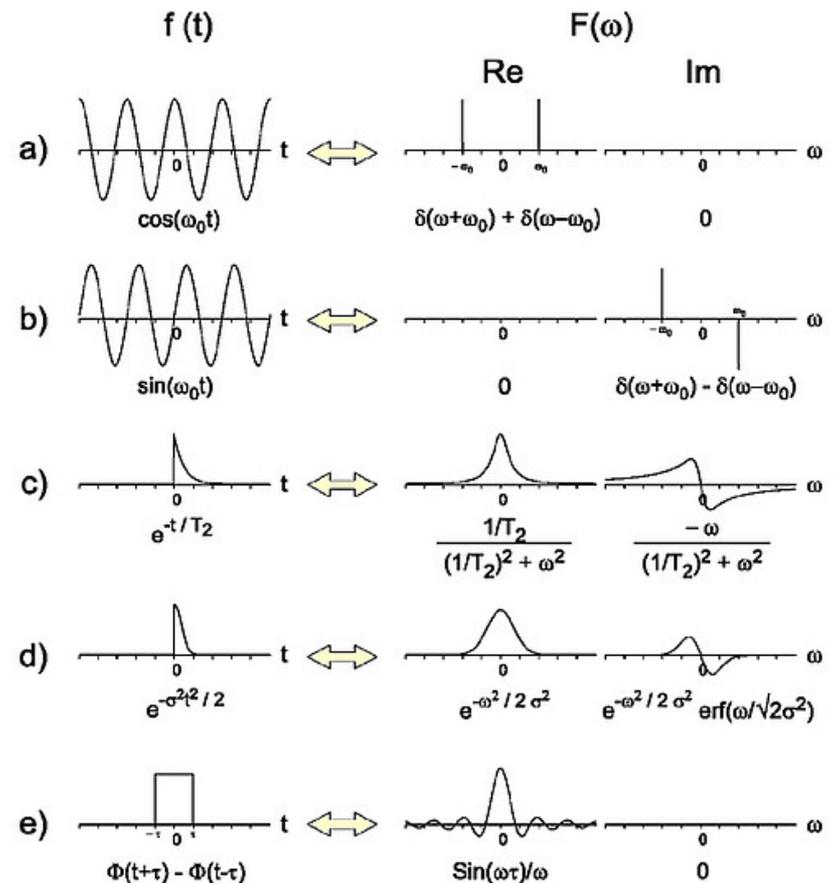
Fourier Transforms

We use the Fourier 'transform' to decompose a signal $x(t)$ into a new function $X(f)$ which is a sum of sines and cosines (or complex sinusoid) of varying amplitude and frequency.

- We talk about the Fourier Transform as the mapping from the signal $x(t)$ (of time) to the signal $X(f)$ (of frequency).
- We call this new function $X(f)$ the frequency spectrum of $x(t)$.
- $X(f)$ is invertible. We can get $x(t)$ back again exactly.
- When we talk about **Frequency** we most likely are always meaning the term as connected with the Fourier mathematics.
- Looking at a signal in terms of frequency often simplifies our understanding of the waveform.
- The results which the Fourier transform yields jive well with empirical real world experiments.

As far as we are concerned, the family of Fourier series/transforms is a very simple concept to understand.

- We really don't need to get into the math at all. We can accept that they are rules.
- For the remainder of the slides, I may use the terms Fourier transform, FFT, DFT, and Spectrum all interchangeably.



Methods for Measurement

Introduction

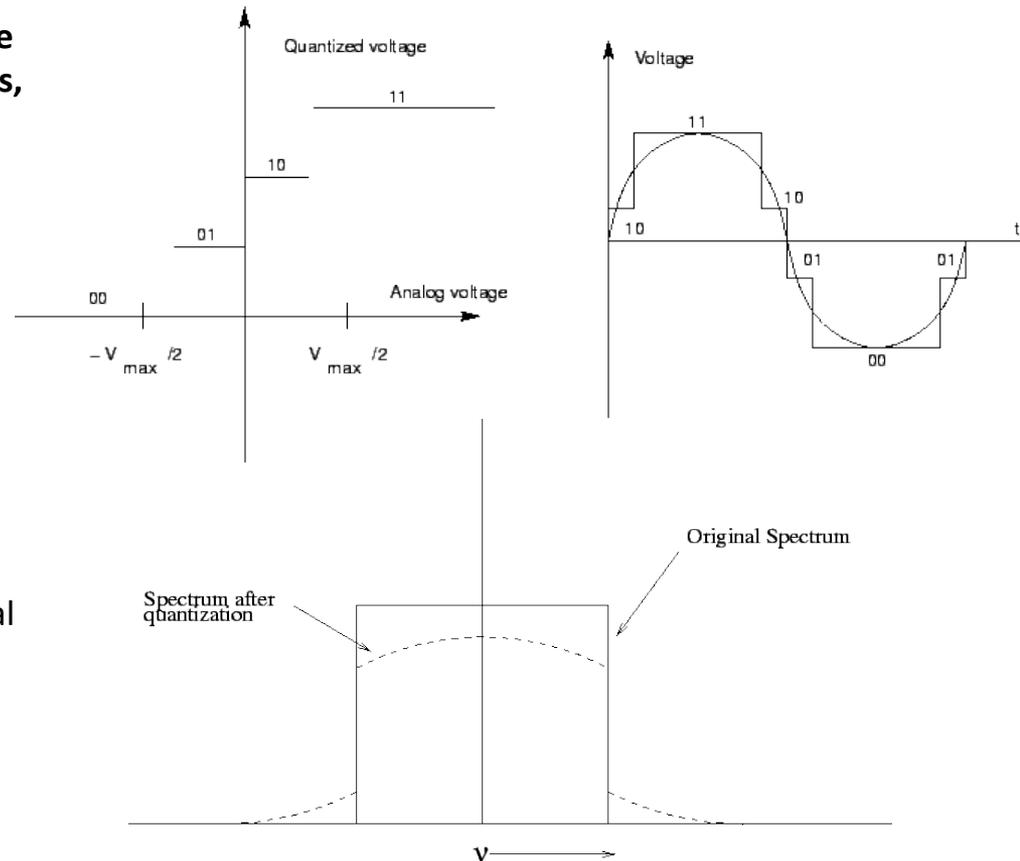
Sampling Basics

*“The assertion made by the Nyquist-Shannon sampling theorem is simple: if you have a signal that is perfectly band limited to a **bandwidth** of f_0 then you can collect all the information there is in that signal by sampling it at discrete times, as long as your sample rate is greater than $2f_0$.” [1]*

We call this property, that the Fourier Transform of the analog signal is bounded on some range of frequencies, finite support. [2]

Unfortunately, in real life there is no such thing as a perfectly band-limited signal, as this would imply that the signal is infinite in duration, or would violate the Time-Bandwidth Product Theorem.

- We will talk more about the TBP later.
- All sampling systems are a compromise where we make best guesses about what the analog signal's spectral content looks like and the amount of dynamic range we need to quantize the amplitude.
- With analog components we try and filter the signal to make it more band-limited.
- We use analog attenuators and amplifiers to make the best use of our quantized range.
- For most systems, our sampled approximation is VERY good. [3]



Methods for Measurement

Introduction

Sampling Basics

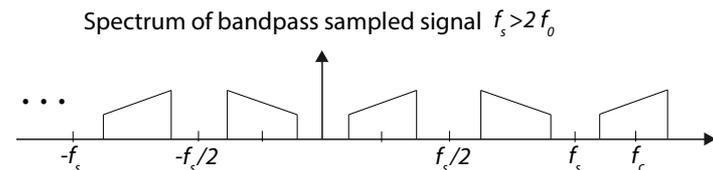
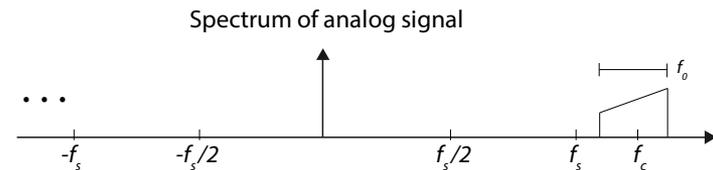
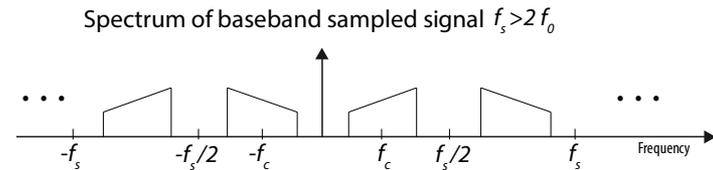
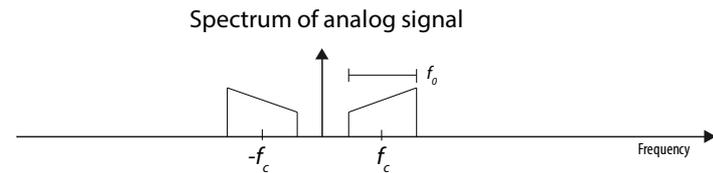
*“The assertion made by the Nyquist-Shannon sampling theorem is simple: if you have a signal that is perfectly band limited to a **bandwidth** of f_0 then you can collect all the information there is in that signal by sampling it at discrete times, as long as your sample rate is greater than $2f_0$.” [1]*

The sampling theorem is about the bandwidth of a signal, not about its center frequency.

- A narrow pulse which repeats at 2Hz may have a bandwidth of 20kHz. Think Cymbals.
- A 500Hz sinewave played through a speaker may have distortion products which extend to 10kHz.

We can sample a signal whose center frequency is above the sampling rate, we call this Band-pass sampling.

- The illustration shows how a signal above f_s if properly band-pass filtered before being sampled will alias down.
- This was/is common in many Software RF applications where ADC sampling rates are inadequate for baseband sampling.



Methods for Measurement

Introduction

Sampling Basics

*“The assertion made by the Nyquist-Shannon sampling theorem is simple: if you have a signal that is perfectly band limited to a **bandwidth** of F_0 then you can collect all the information there is in that signal by sampling it at discrete times, as long as your sample rate is greater than $2F_0$.” [1]*

The sampling theorem doesn't say that the samples you view on the computer screen are going to look like the analog signal.

- It says that there is a reversible mathematical process. We can get our analog signal back again. [2]
- The computer plotting software must choose some way to connect the samples (dots).
- If we want the signal to look close to what we are used to seeing on a analog oscilloscope, we need to oversample the analog signal.

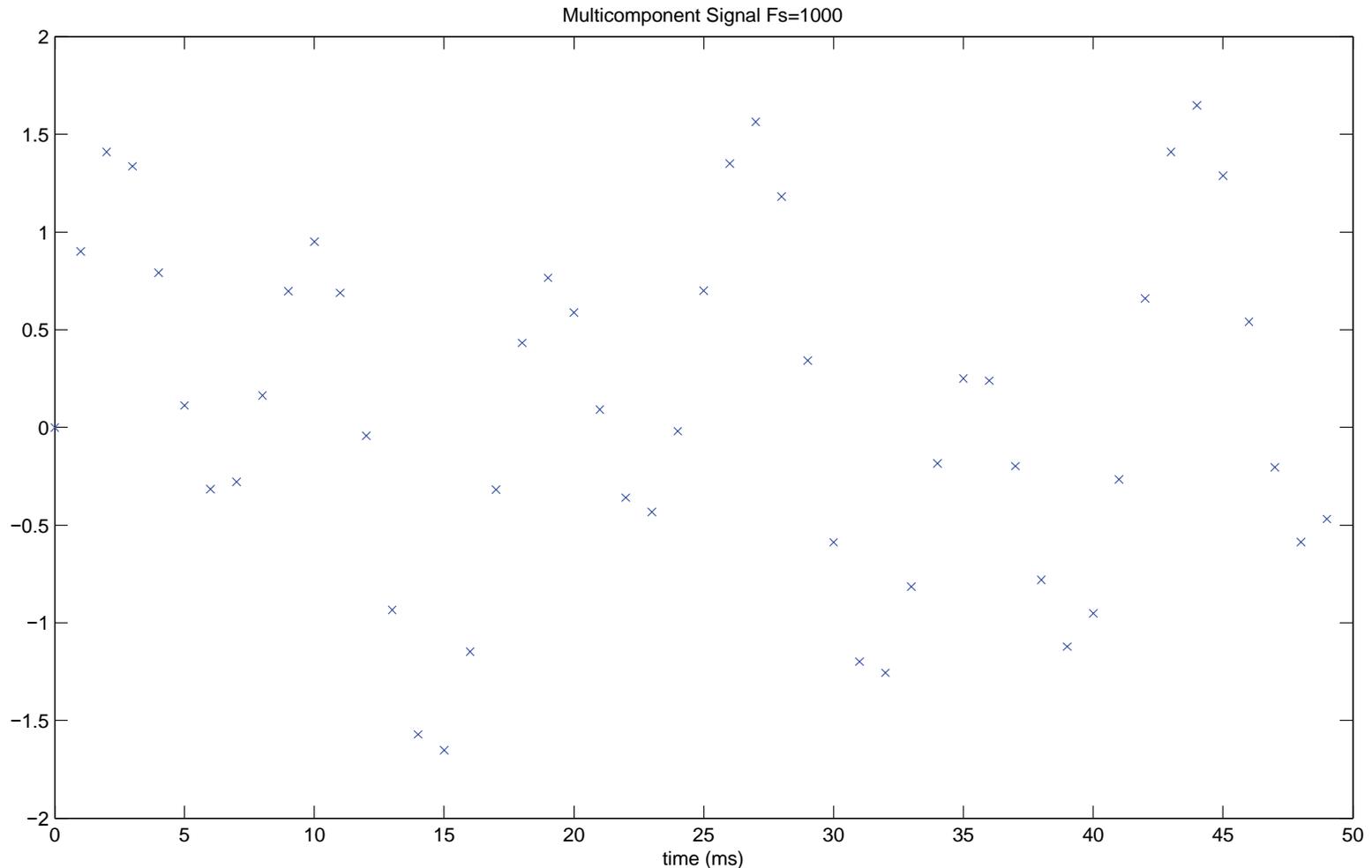
Example

- We sample two superimposed sinusoidal signals: 50Hz and 120Hz
- Analog Function: $x(t) = 0.7 \cdot \sin(2 \cdot \pi \cdot 50 \cdot t) + \sin(2 \cdot \pi \cdot 120 \cdot t)$;
- Sampled at 1kHz and 10kHz with 64bit double precision floating point.

Methods for Measurement

Introduction

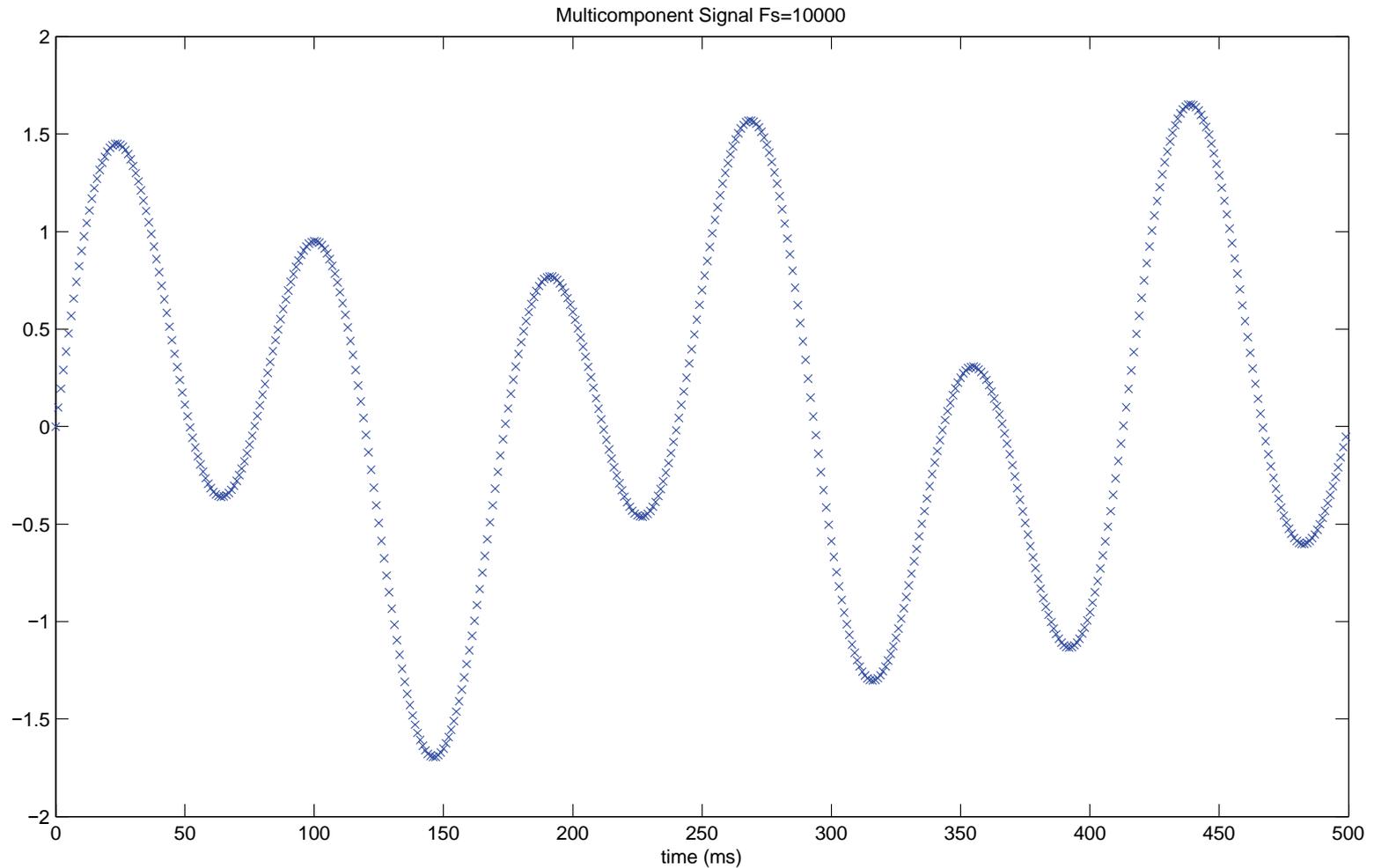
Sampling Basics



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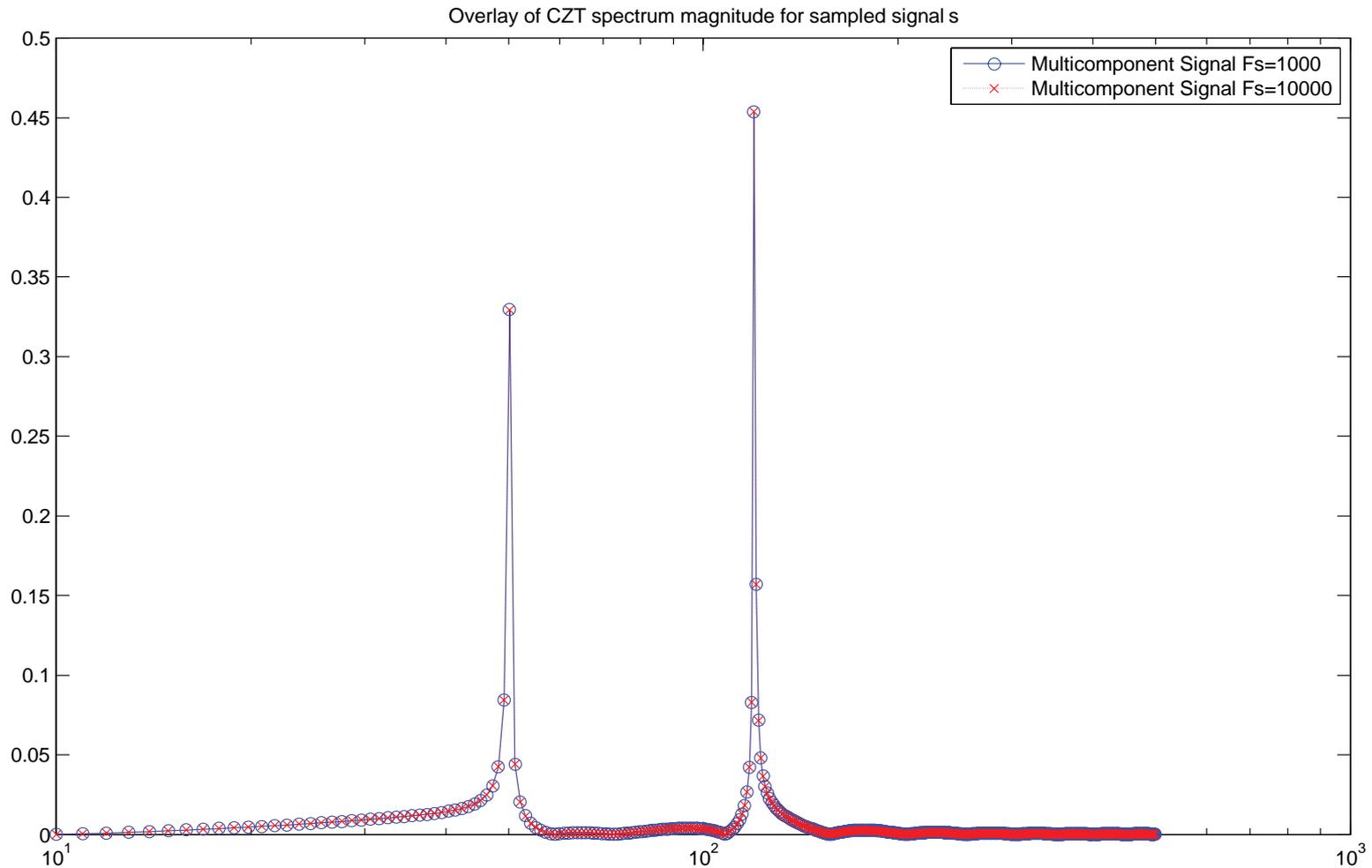
Sampling Basics



Methods for Measurement

Introduction

Sampling Basics



Methods for Measurement

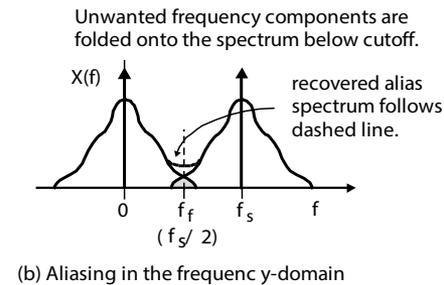
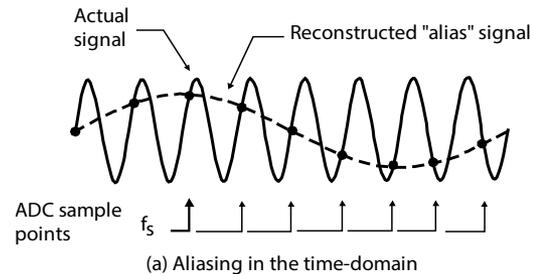
Introduction

Illustrations thanks to Agilent [3]

Analog Basics – Baseband Sampling / Filtering

We use analog filters to reduce aliasing and DC offset.

- Low order fixed AC coupling HP analog filter to protect ADC from DC offset.
- Low Order fixed analog LPF to protect ADC from aliasing.
- Common to have digital FIR LPF integrated in ADC with higher order roll off.
- Advanced ADC may allow custom FIR filter or variable cutoff frequency.
- We must take these filters into account when working with measurement systems, DSPs, mixing consoles, and etc. in many circumstances. Typically 20-100 samples delay for Digital FIR.

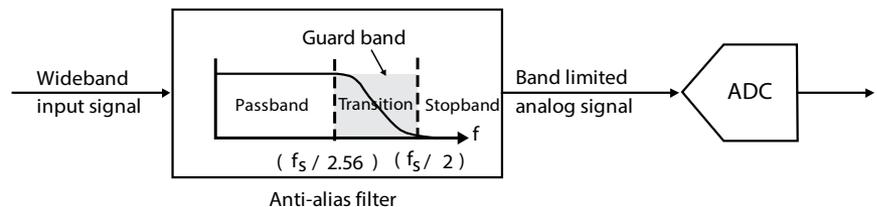


$$f_s > 2 (f_{\max})$$

where

f_s = sample rate

f_{\max} = highest frequency component



Methods for Measurement

Introduction

Questions / Additional Items / References

- Wait, I said I wasn't going to talk about sampling!!! In conclusion, it just works for all practical audio applications.
- We skipped a LOT of details, including DACs and reconstruction, I would recommend the references [2] and [3] for more information.
- We will go over what Spectrum and Bandwidth really mean in the section on Spectrum Analyzers.

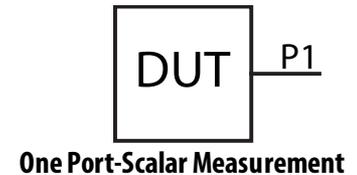
[1] T. Wescott, "Sampling: What Nyquist Didn't Say, and What to Do About It," Wescott Design Services, Oregon City, OR, 2010.

[2] R. Allen and D. Mills, "Signal Analysis: Time, Frequency, Scale and Structure," IEEE Press, Piscataway, NJ, 2004.

[3] R. Ziemer, W. Tranter, and D. Fannin, "Signals and Systems: Continuous and Discrete, 4th Edition," Prentice Hall, Upper Saddle River, NJ, 1998.

Methods for Measurement

Digital SPL Meter



- A calibrated microphone samples the ambient sound in the room.
- Really not any different than a handheld multi-meter.
- RMS-Peak mode, different ranges on analog models, etc.
- Select Different industry standard input filters. A,C, etc.
- Implementing on a computer has the advantage of longer or more complex averaging and filtering.

Sample Blocks of N values from the microphone at Fs Hz

- N is normally set to be at least 2.5x the length of the lowest frequency of interest and Fs is set to 2x + 10% the highest frequency of interest.

Each Block is processed in the time domain

- Each block is filtered according to the industry standard
- The RMS,peak, or etc. value of the block is computed

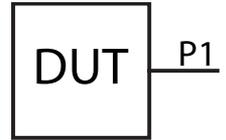
Successive Blocks are averaged

- There are lots of way in which this is done. Overlapping, etc.
- This is where all the magic happens. [9]



Methods for Measurement

SPL Meter



One Port-Scalar Measurement

A SPL Meter should be used for:

- Testing for compliance with local laws
- Testing to make sure you are not damaging your ears

A SPL Meter should **NOT** be used for:

Measuring the sensitivity of a loudspeaker

- Why Not?
- This one number tells us nothing about the loudspeaker unless it is localized in frequency. And even at that point, who cares about one frequency.

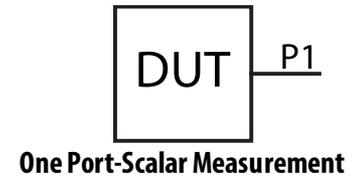
Measuring the frequency response of a loudspeaker in conjunction with a sine wave generator.

- We will see with the spectrum analyzer example how we get skewed results based on the frame size.



Methods for Measurement

Oscilloscope



We could extend our SPL meter example a little further and display the real-time recorded time series on the computer screen.

- We could also just connect our microphone up to our digital oscilloscope.
- As blocks of N samples are recorded by the ADC, they are scrolled at a pleasing rate, within the limits of buffering, across the screen.
- The graticules on the screen are calibrated for voltage and sampling rate.

An oscilloscope is a device which allows the observation of a signal as it varies in time.

For us to view the signal the oscilloscope must oversample the bandwidth we are interested in.

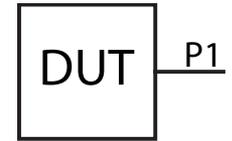
The model of an analog signal is infinite in duration. We must choose some finite amount that we are going to look at. A screen full, the amount of fast RAM, etc.

Simply viewing the waveform is not very accurate for measurement, we use statistical methods to gain insights into the signal.



Methods for Measurement

Spectrum Analyzer / RTA



One Port-Scalar Measurement

Like an oscilloscope, a typical spectrum analyzer is not a device designed to give high accuracy measurements. It is designed to assist the user in visualizing signals in real-time so he/she can draw conclusions.

Transducers

- A calibrated pressure/random incidence microphone samples the sound field in the room.
- An accelerometer measures box vibration.

Uses

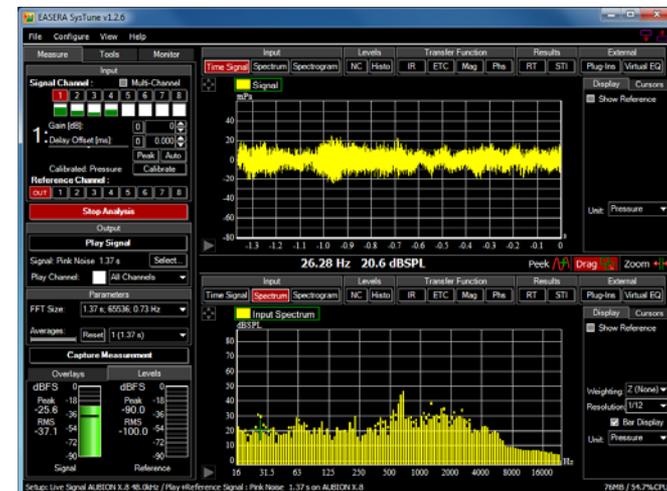
- For measuring steady state spectral content of loaded room [1]. Loudspeaker + Environment.
- Averaging over many randomly distributed microphones yields the best results [2].
- Identify enclosure resonances and eliminate bracing issues.



Other Stuff

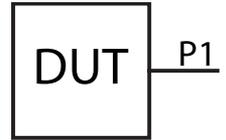
- Older stand alone RTAs are normally weighted to correct for a pink noise stimulus, watch out.
- Not to be confused with a RF 'spectrum analyzer' they don't work the same way!
- We are sampling at base-band.
- We also normally look at and talk about the Magnitude Spectrum in the audio world.

Next- some background. Most of the information in this section applies to all the sections to come.



Methods for Measurement

Spectrum Analyzer / RTA



One Port-Scalar Measurement

History

“The Spectrum can be an efficient, convenient, and often revealing description of [a] function.” –Garner [9]

Spectral Analysis is the decomposition (using the Fourier Transform/Series) of a time series into a collection of weighted sinusoidal functions of different frequencies, we call spectral components.

Fourier Analysis originated in two separate fields of study, acoustical/optical waves, and astronomical and geophysical periodicities.[9]

- Motions of the planets, tides, and weather.
- Connection to the vibrating string problem and behavior of light waves.
- Gardner in [9] cites Lagrange and Euler as first to work with Spectral Analysis.

Sir Arthur Schuster (1851-1934) invented the *periodogram* around 1900 to search for hidden periodic behavior in random data.

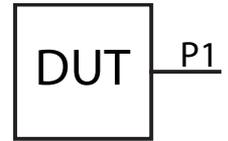
- The periodogram is simply the squared magnitude of the Fourier transform of a finite set of data.
- Not much has changed except that computers can now do this in real-time.



Methods for Measurement

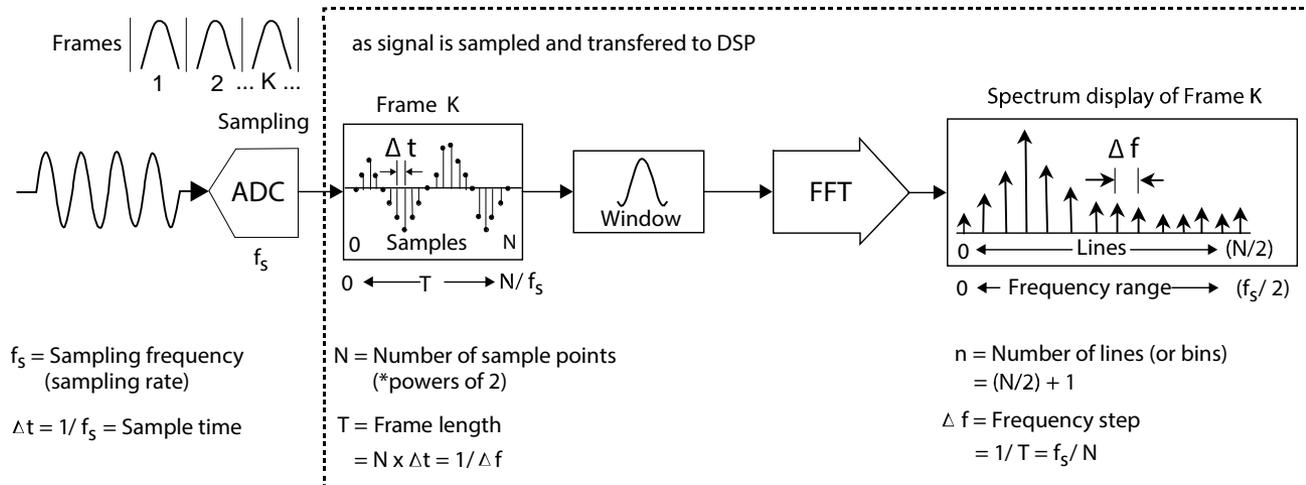
Spectrum Analyzer / RTA

Illustrations thanks to Agilent [3]



One Port-Scalar Measurement

Typical Baseband Direct Sampling Spectrum Analyzer Block Diagram



Assume we have a band-limited sampled signal received by the host PC from our ADC.

- We would like to view the magnitude spectrum of the sound picked up by the microphone in a continuous fashion.

The continuous stream of samples must be divided into time processing frames by windowing out N samples.

- Why? Well computers are finite devices.
- Doing nothing, is what we call a rectangular window.
- Other predefined windows: Bartlett, Hamming, Hanning, Blackman, Kaiser, and etc.

FFT each windowed processing frame.

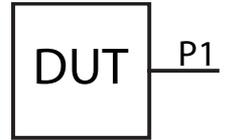
- This gives us the frequency content of the samples and window combined as complex values.
- Find the magnitude or power.

Choose how we update the user interface.

- The simplest would be replacement or averaging.
- Many methods exist. Overlap, etc.
- This may be integrated with the windowing, FFT operation in more complex designs.

Methods for Measurement

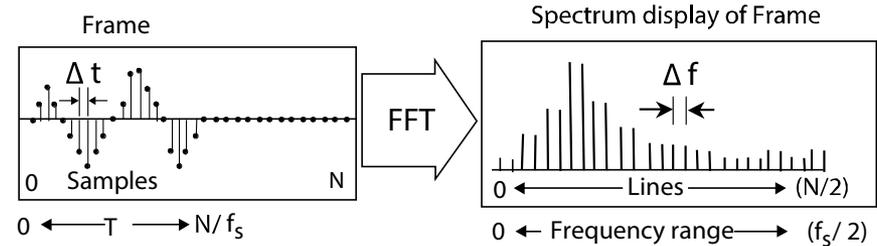
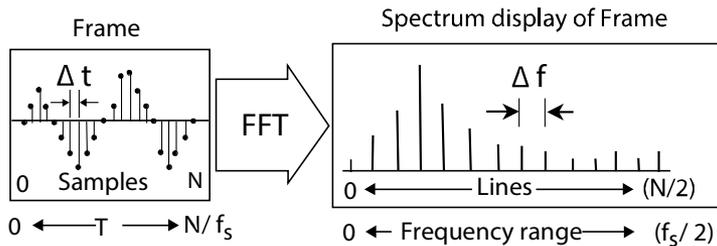
Spectrum Analyzer / RTA



One Port-Scalar Measurement

Windowing, Uncertainty Principles, and the FFT

Time-Bandwidth Product Theorem



The Fast Fourier Transform (FFT) is a computer algorithm which maps a sequence over time to a sequence over frequency [5].

- Fast implementation of Discrete Fourier Transform.
- Because this is a computer, the FFT can only operate on a finite number of samples.
- The theory behind the FFT assumes that the signal is **periodic**.
- F_{max} is $F_s/2$ as discussed in the Introduction.
- The output of the FFT for real input signals is $(N/2) + 1$ uniformly spaced complex frequency points from DC to F_{max} .

The resolution of the FFT frequency grid is a fixed function of the sampling rate and frame length. $\Delta f = F_s / N$

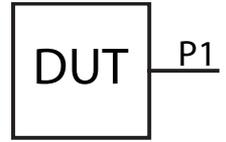
- We call the frequencies on which these points fall the FFT bins.
- Generally the peaks may not be in the correct location or amplitude because of the grid spacing.
- In theory, we could increase the resolution by just adding zeros to the frame, making N larger.
- Don't confuse the FFT grid resolution relationship with "The Uncertainty Principle."

This Resolution relationship is only true for the FFT/DFT.

- The Chirp-Z transform has an arbitrary number of uniformly spaced frequency bins [1].
- The NUFFT has an arbitrary number of non-uniformly spaced frequency bins [4].
- Why don't we use them? They are slow(er).

Methods for Measurement

Spectrum Analyzer / RTA



One Port-Scalar Measurement

Windowing, Uncertainty Principles, and the FFT

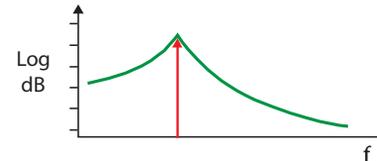
Time-Bandwidth Product Theorem

We can say generally, that the Window is the most important part of this type of Spectrum Analyzer.

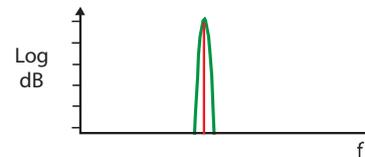
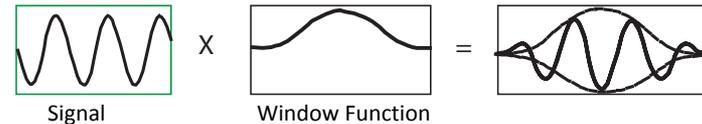
- We must choose some frame of N of samples that we are going to transform with the FFT.
- The window is some function we perform on the frame to make the samples it contains more periodic.
- We look through this window to try and measure the global signal.
- In a Spectrum analyzer there is generally no way to make sure the frame falls where it 'should' on the signal, that is why we window.
- Modern analyzers employ advanced techniques to move the frame around the signal, resize the frame, reshape the window function and combine the results, all as fast as possible.
- When we choose a poor window, the FFT is not wrong, the **measurement** is wrong.

Examples

- The samples of the (red) frame *happen* to be periodic in the frame. The FFT bin shows its exact frequency and amplitude.
- The samples of the (green) frame are not periodic in the frame. The energy has leaked out of the bin, giving an incorrect measurement result.
- If we apply a window function to the (green) frame which makes it periodic, our accuracy is greatly improved.



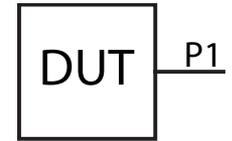
(red) – Signal In FFT Bin
(green) – Spectral Leakage



Spectrum of Windowed leaky frame. Accuracy has improved significantly.

Methods for Measurement

Spectrum Analyzer / RTA



One Port-Scalar Measurement

Windowing, Uncertainty Principles, and the FFT

Time-Bandwidth Product Theorem

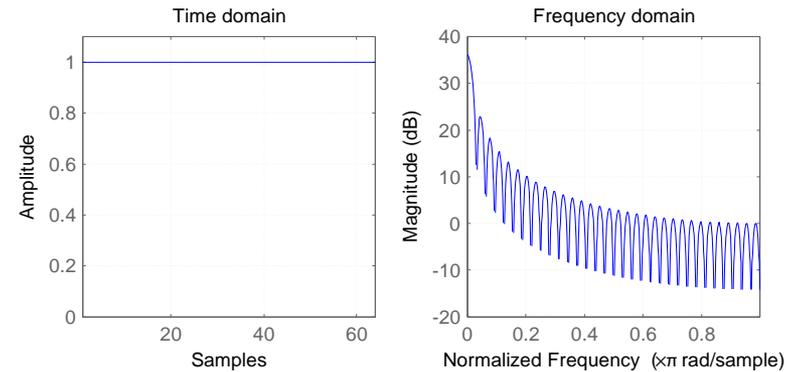
Qualities of the window in both the time domain and the frequency domain.

The Window Function

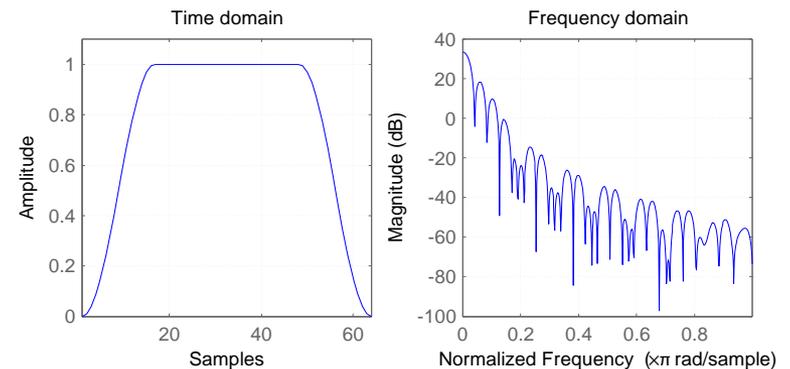
- As the length of the window/frame is increased, the width of the main-lobe decreases and the side-lobe attenuation increases marginally.
- As the amount of taper increases, the width of the main-lobe increases, and the side-lobe attenuation increases.

When combined with our Frame and Transformed

- The resolution bandwidth is primarily influenced by the width of the window function main-lobe.
- The amplitude accuracy is influenced primarily by the relative level of the window functions main-lobe to the side-lobes.



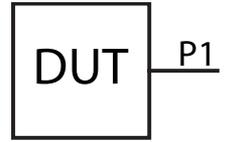
Rectangular Window – Narrow Mainlobe



Tukey Window – More Taper

Methods for Measurement

Spectrum Analyzer / RTA



One Port-Scalar Measurement

Windowing, Uncertainty Principles, and the FFT

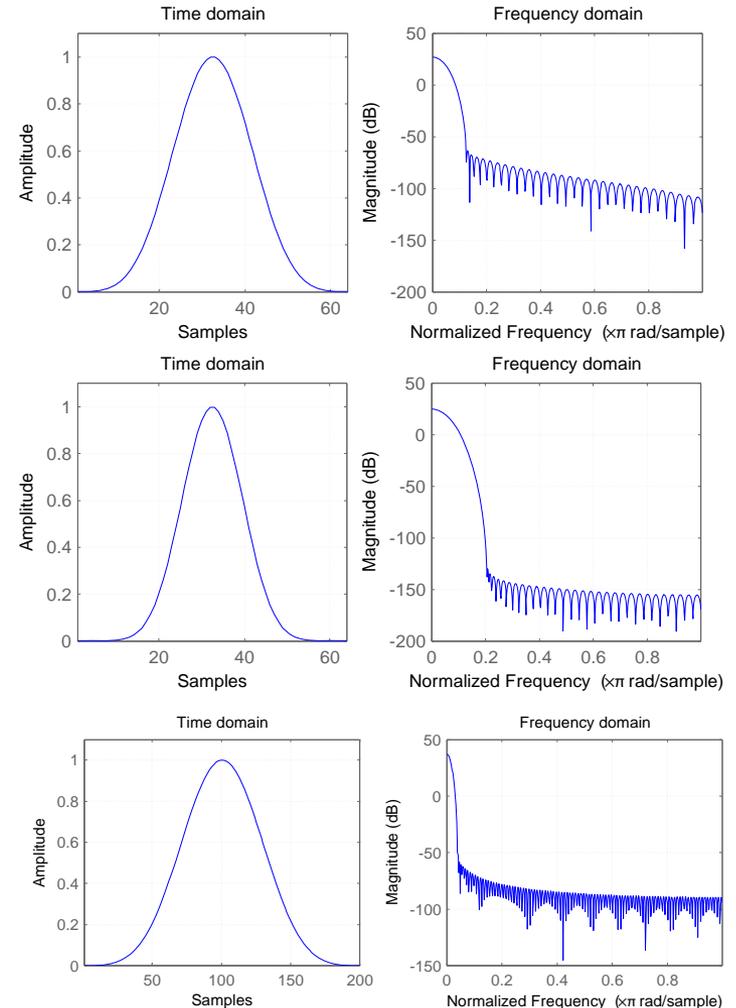
Time-Bandwidth Product Theorem

The Kaiser Window

- Two parameters control length L and taper β .
- Schafer [5] gives closed form solutions to find L , β , and a window function given desired main-lobe width and side-lobe attenuation.

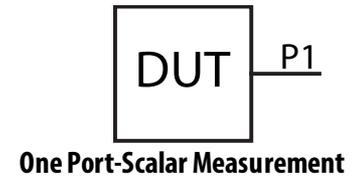
When using a Spectrum Analyzer we are normally not interested in that frame alone; but an estimation of the frequency content of the global analog signal which extends over many frames.

- We must choose windows which tradeoff amplitude accuracy/reduced leakage and frequency accuracy/smearing.
- This will vary significantly depending on what type of signal we are measuring. You need to setup your equipment and experiment.
- So what is it that causes this smearing in the frequency domain? The **Time-Bandwidth Product Theorem**.



Methods for Measurement

Spectrum Analyzer / RTA



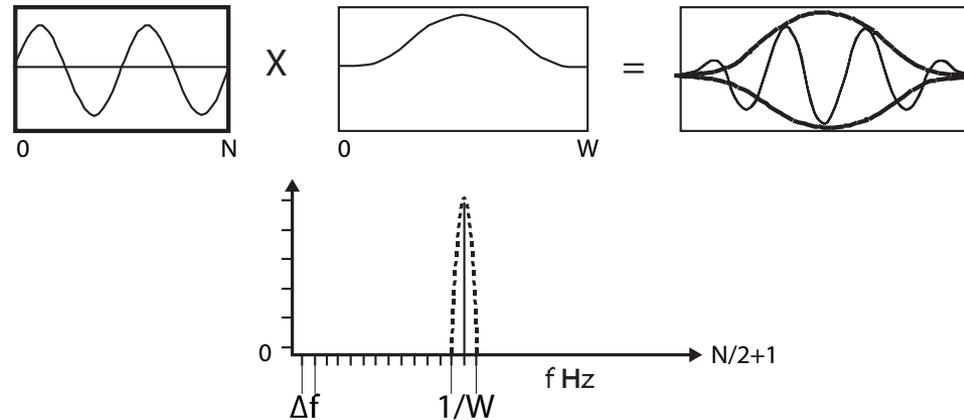
Windowing, ~~Uncertainty Principles~~, and the FFT

Time-Bandwidth Product Theorem

“The use of the word ‘uncertainty’ is a misnomer, for there is nothing uncertain about the ‘uncertainty relation’ ...” –Skolnik [7]

The Time-Bandwidth product theorem is a fundamental statement about Fourier Transform Pairs [6]

- “If the effective bandwidth of a signal is W , then the effective duration cannot be less than about $1/W$.” [8]
- If the effective duration of a signal is N , then the effective bandwidth cannot be less than about $1/N$.
- If something is broad in one domain it will be narrow in the other domain.
- Does NOT say that it is impossible measure Time and Frequency simultaneously.



We switch from using the rectangular window of duration N to some window of duration $W < N$ to reduce spectral leakage.

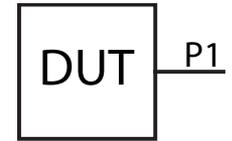
- Application of Time-Bandwidth Theorem. We are going to skip all the math and proofs.
- $1/N \ll 1/W$ - Our frequency selectivity is reduced (smeared) from $1/N$ to $1/W$
- The resolution of frequency grid points in the FFT has not changed.

This is a statement about the Fourier Transform

- If we are going to use the Fourier Transform to break up our original signal into set of spectral components, then we have to live with it.

Methods for Measurement

Spectrum Analyzer / RTA



One Port-Scalar Measurement

Summary of a typical Spectrum Analyzer

The typical Baseband Direct Sampling Spectrum Analyzer is a complex measurement device with parameters which must be setup correctly by the user to accurately measure a signal.

The Resolution Bandwidth of a Spectrum Analyzer is a function of

- The sampling rate of hardware
- The frame size
- Width of Window function main-lobe.

The amplitude accuracy is a function of

- Dynamic Range, SNR, ENOB of hardware.
- Relative amplitude of main-lobe to side-lobe of window.

Like an oscilloscope, a typical spectrum analyzer is not a device designed to give high accuracy measurements. It is designed to assist the user in visualizing signals in real-time so he/she can draw conclusions.

We can play a stimulus with known statistical distribution through our system.

- We play pink noise through the system, which will excite all the frequencies of interest.
- We record and average the spectrum from a number of microphones around the room.[2]
- Find a compromise set of EQ points to reduce resonances in the room.

We can use the spectrum analyzer to view the spectral content of live/recorded music.

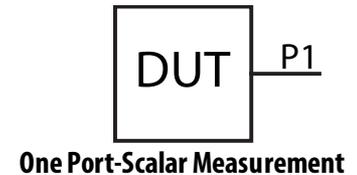
- Assist our ears in placing EQ points to cleanup the playback.

We can use the spectrum analyzer to view the spectral content an accelerometer attached to the panel of a cabinet.

- Hit the box with a impulse hammer, or just trigger on the level. Observe the resonant modes of the box.
- Help decide where additional bracing should go.

Methods for Measurement

Power Spectrum / RTA



Questions / Additional Items / References

This has been a lot of background information to cover.

- [1] S. Stearns and R. David, “Signal Processing Algorithms in Fortran and C,” Prentice Hall, Englewood Cliffs, NJ 1993.
- [2] G. Cengarle and T. Mateos, “Effect of Microphone Number and Positioning on the Average of Frequency Responses in Cinema Calibration,” presented at the AES 136th convention, Berlin, Germany, 2014 April 26–29.
- [3] Agilent, “Vector Signal Analysis Basics, Application Note 150-15,” Palo Alto, CA, 2004.
- [4] June-Yub Lee and Leslie Greengard, “The type 3 Nonuniform FFT and its Applications,” J. Comput. Phys. 206, 1–5, 2005.
- [5] A. Oppenheim and R. Schaffer, “Discrete-Time Signal Processing, Second Edition,” Prentice Hall, Englewood Cliffs, NJ, 1999.
- [6] L. Cohen, “Time-Frequency Analysis,” Prentice Hall, Upper Saddle River, NJ 1995.
- [7] M. Skolnik, “Introduction to Radar Systems,” McGraw-Hill Book Co., 1980
- [8] M. Ackroyd, “ Instantaneous and time-varying spectra- An Introduction,” Radio Electronics Engineering, vol. 239, pp. 145-152, 1970.
- [9] W. Gardner, “Statistical Spectral Analysis: A Nonprobabilistic Theory,” Prentice Hall, Englewood Cliffs, NJ, 1988.

Chapter 2

Methods for Audio Measurements

One Port

- SPL Meter
- Spectrum Analyzer / RTA

Two Port – Transfer Function

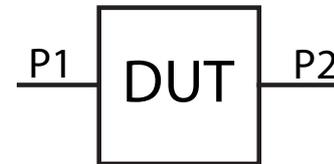
- Direct Form
- Matched Filtering

Device Under Test (DUT)



One Port-Scalar Measurement

The DUT does something and we measure it.

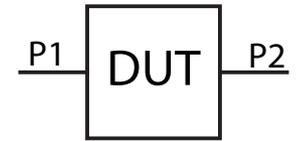


Two Port-Vector Measurement

We input something to the DUT on P1 and measure what the DUT does to it on P2

Methods for Measurement

Transfer Functions – Direct Form



Two Port-Vector Measurement

Motivating Example – A Loudspeaker

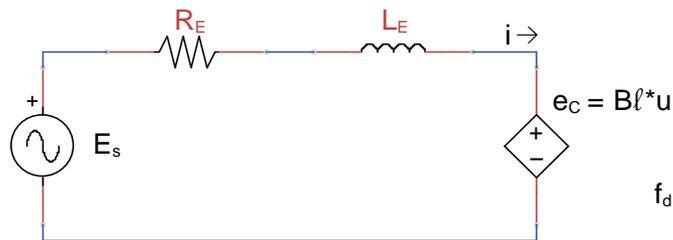
- For all practical purposes we could choose any part of the typical pro-audio signal path.

A Loudspeaker is an electro-mechanical device which produces acoustic output proportional to the current applied to it.

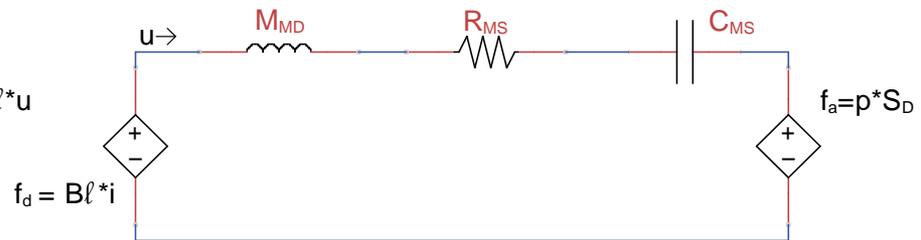
We would like to find a measurement which characterizes the speaker as fully as possible, and we would like this measurement to have a good correspondence with our perceived qualities of it.

As we discussed in the introduction, we must first develop a model of the loudspeaker.

- An industry accepted model of a loudspeaker is the Small Signal Model [2].
- The circuit below shows a simplified schematic [3].
- Z_a , the acoustic impedances will be determined by the box, room, etc.
- More accurate loudspeaker models add more lumped components or dependencies (time, frequency, amplitude).

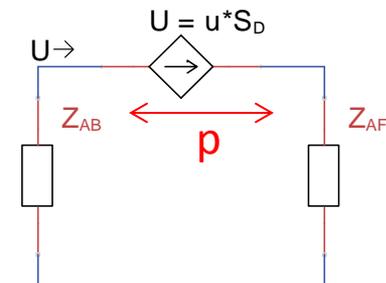


E_s - Amplifier Voltage
 R_e - Resistance
 L_e - Inductance
 i - Current



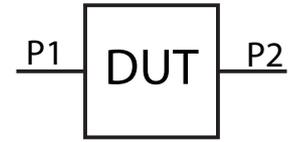
Z - Acoustical Imp
 C - Compliance
 R - Damping
 M - Mass

f - Force
 u - Cone Velocity
 U - Acoustic Volume Velocity
 p - Acoustic Pressure



Methods for Measurement

Transfer Functions – Direct Form



Two Port-Vector Measurement

The multi-domain schematic represents a system of linear differential equations.

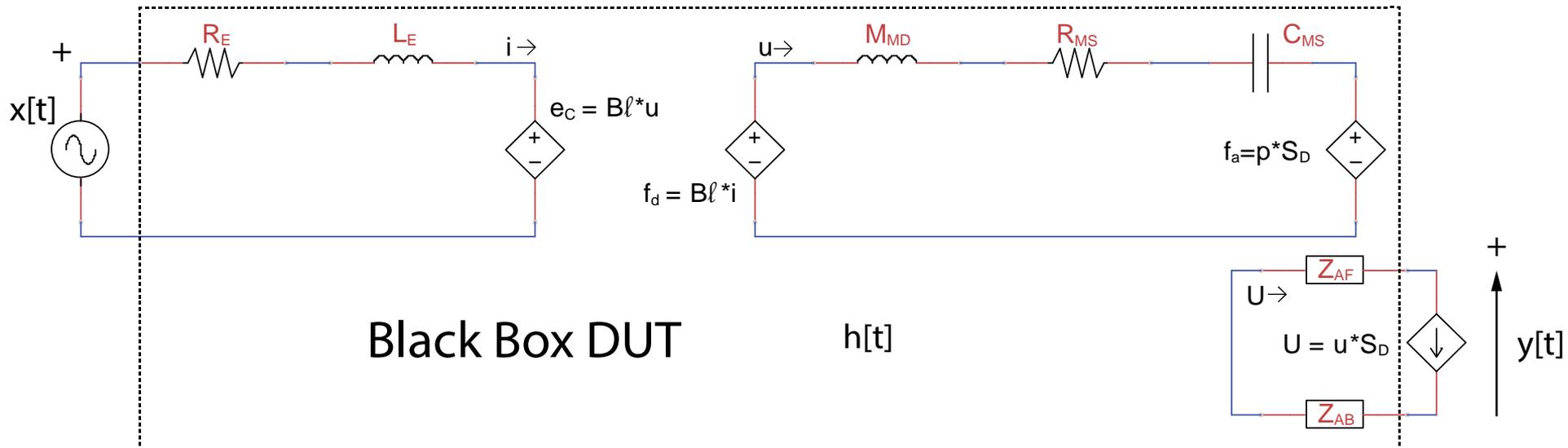
- We will simply use it as a guide, not determine exact solutions.

We will use the transfer function from the voltage input $x[t]$ to the pressure output $y[t]$ to characterize the DUT.

- We will treat everything between these two points as “black box”, later denoted $h[t]$.

- We will provide a known voltage stimulus $x[t]$.
- We will record the SPL at a known distance and impedance condition $y[t]$.
- We will use digital signal processing techniques to determine $h[t]$ and spectral analysis to visualize the system.

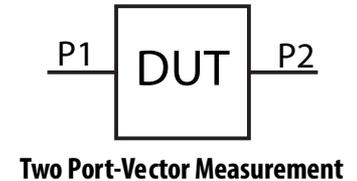
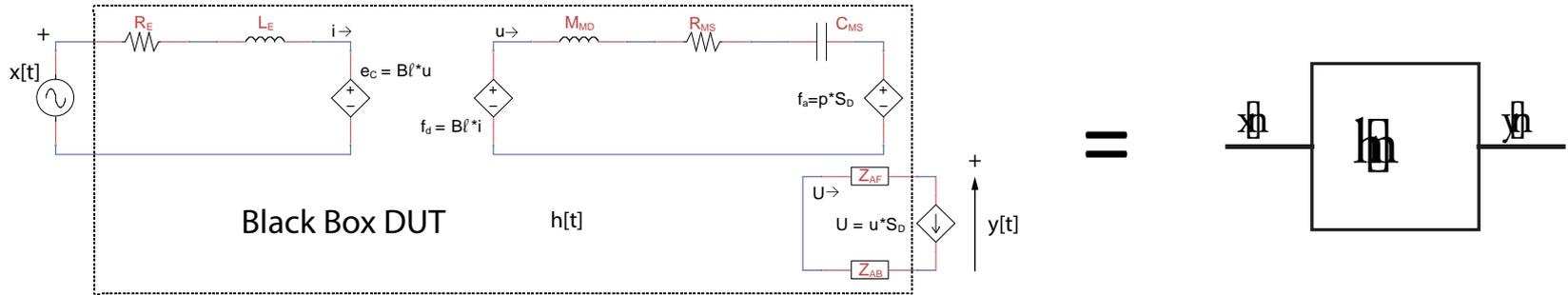
If we wanted to determine the individual values (functions) in the model, the process would be much more involved.



Methods for Measurement

Transfer Functions – Direct Form

ASSUME THE MODEL IS TRUE



Our model describes the physical system we are trying to measure. But when we do blind measurements, they are not really connected to the model

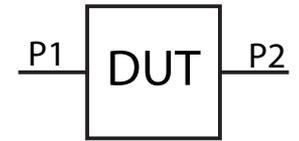
- Although we have the model, we are just assuming that reflects how our speaker works. If that assumption is true, then we can be confident, that when we perform the measurement, the results we get represent something about the physical system.
- More complex measurement software, such as programs for determining Thiele-Small parameters, will integrate the model into the software. We will talk a little more about this later, but most pro-audio software is not model assisted.

Under this assumption:

- Some very neat properties of LTI systems and the Fourier Transform then allow us to uniquely determine the 'Transfer Function' of the black box DUT.
- If the assumption is false, then our solutions may not be unique or really represent what we are trying to measure.

Methods for Measurement

Transfer Functions – Direct Form



Two Port-Vector Measurement

So, how do we turn our Spectrum Analyzer into a device which can take an accurate measurement of a black-box DUT?

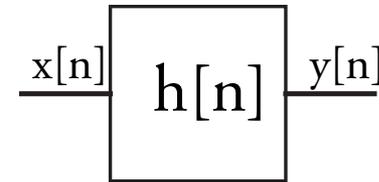
- We put constraints on the DUT and the stimulus.
- We more strictly define our Window, and FFT length.
- We average multiple measurements.

Linear Time Invariant systems and decomposition into sine waves. The basic math is pretty straight forward.

- $h[n]$ is the impulse response of our LTI system.
- $x[n]$ is our stimulus signal
- $y[n]$ is our measured response
- $y[n] = T\{x[n]\} = h[n]*x[n]$ is our LTI convolution system

If $h[n]$, our DUT is LTI, then the following are true

1. *Superposition*
 - Scaling: If you scale x by a then you scale y by a
 $T\{ax[n]\} = aT\{x[n]\} = ay[n]$
 - Additive: Two components don't interfere with each other
 $T\{x_1[n] + x_2[n]\} = T\{x_1[n]\} + T\{x_2[n]\} = y_1[n] + y_2[n]$
2. *Shift Invariance*: a time shift or delay on the input corresponds to time shift or delay on the output.



$\mathcal{F}\{\cdot\}$ Fourier Transform

$$X[f] = \mathcal{F}\{x[n]\}$$

$$Y[f] = \mathcal{F}\{y[n]\}$$

$$H[f] = \mathcal{F}\{h[n]\}$$

\otimes Convolution

$$y[n] = h[n] \otimes x[n]$$

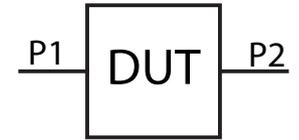
$$y[n] = \sum_k x[n] h[n-k]$$

$$Y[f] = H[f]X[f]$$

$$y[n] = \mathcal{F}^{-1}\{H[f]X[f]\}$$

Methods for Measurement

Transfer Functions – Direct Form



Two Port-Vector Measurement

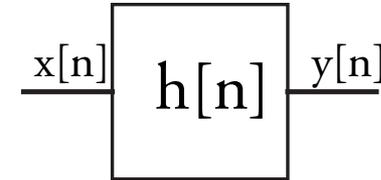
Also, the only functions x and y which satisfies all the conditions are complex sinusoids or a sum of complex sinusoids.

The idea of decomposition into sinewaves (Spectral Analysis) and the type of system it is applicable to, a LTI system, are intimately connected by the definition of the model, they are not independent.

Only a LTI system can be fully characterized by using Spectral Analysis. Luckily, most of the time, this is sufficient to learn something meaningful about the system.

Any LTI system can be uniquely and fully characterized by the rational expression bellow.

$$H[f] = \frac{\sum_{k=0}^M b_k e^{-j2\pi f k}}{\sum_{k=0}^N a_k e^{-2\pi f k}}$$

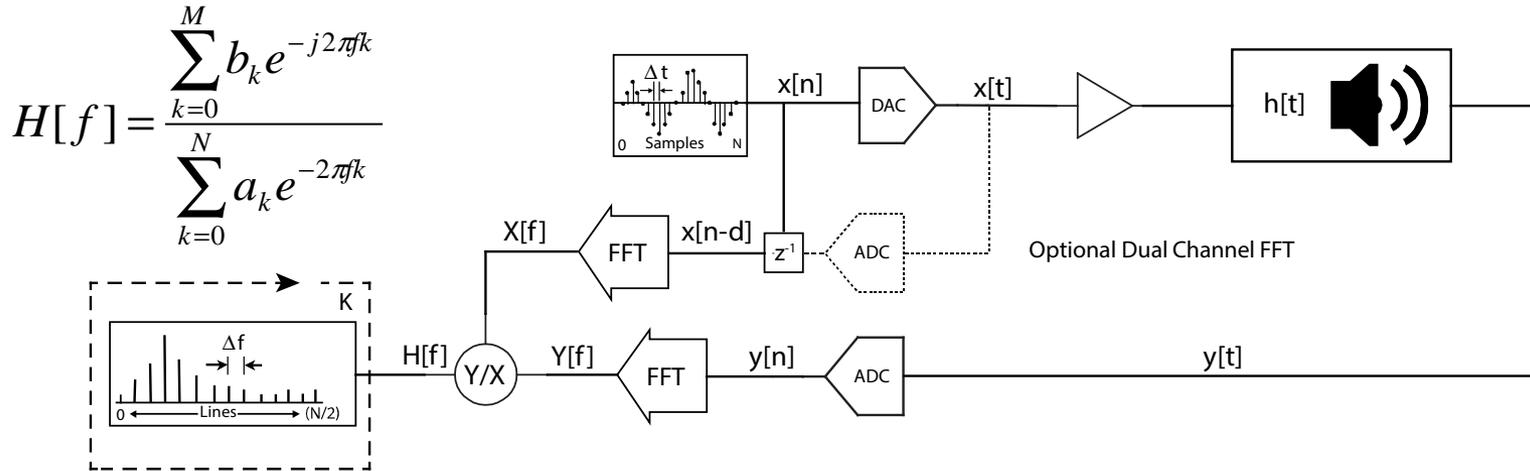
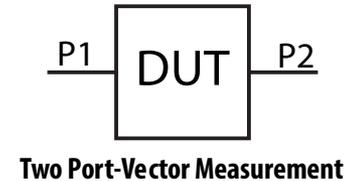


$$\begin{aligned} \mathcal{F}\{\cdot\} & \text{ Fourier Transform} \\ X[f] &= \mathcal{F}\{x[n]\} \\ Y[f] &= \mathcal{F}\{y[n]\} \\ H[f] &= \mathcal{F}\{h[n]\} \end{aligned}$$

$$\begin{aligned} \otimes & \text{ Convolution} \\ y[n] &= h[n] \otimes x[n] \\ y[n] &= \sum_{k=0}^n x[k] h[n-k] \\ Y[f] &= H[f] X[f] \\ y[n] &= \mathcal{F}^{-1}\{H[f] X[f]\} \end{aligned}$$

Methods for Measurement

Transfer Functions – Direct Form



If $y[n] = T\{x[n]\} = h[n]*x[n]$ is our LTI convolution system

A naive calculation of $H[f]$ is really as simple as it looks!

And this is how it is implemented in many of the most basic measurement systems.

What can go wrong here?

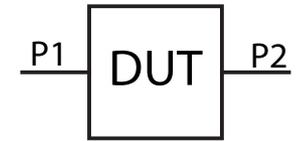
- Delay / Windowing
- Noise (random, impulse)/Reflections/Non-linearities
- Division can get numerically ugly!

What choices does the end-user need?

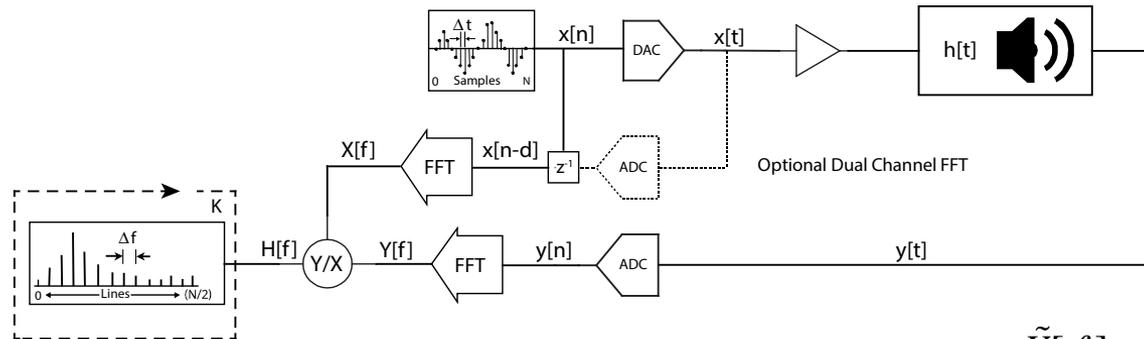
- F_s – The Sampling Rate
- N – The Number of Samples per Frame
- D – The delay to align stimulus and response
- W – The Window to make the frame periodic
- K – The number of frames to average
- The method of averaging frames...
- What will our stimulus be?

Methods for Measurement

Transfer Functions – Direct Form



Two Port-Vector Measurement



Enter Statistical Spectral Analysis

- For implementation of the Direct Form transfer function measurement, the method we use to average frames is the most important choice.
- We would like to choose a method which converges to the actual transfer function as quickly as possible.
- We also need to keep spectral leakage, TBP and noise in mind.

The simplest case for the Magnitude Response does work very well in practice

- We can easily choose a stimulus whose limit spectrum is a constant value such as White Noise

$$\tilde{Y}[f] = |\tilde{H}[f]|^2 \tilde{X}[f]$$

$$\tilde{X}[f] \rightarrow N_0 \text{ as } K \rightarrow \infty$$

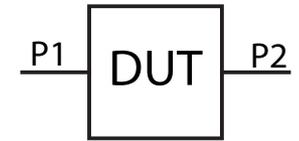
$$|\tilde{H}[f]|^2 = \frac{1}{N_0} \tilde{Y}[f]$$

The Wiener relation says that from sufficiently averaged spectrums X, Y we can determine the magnitude of the transfer function H of the system.

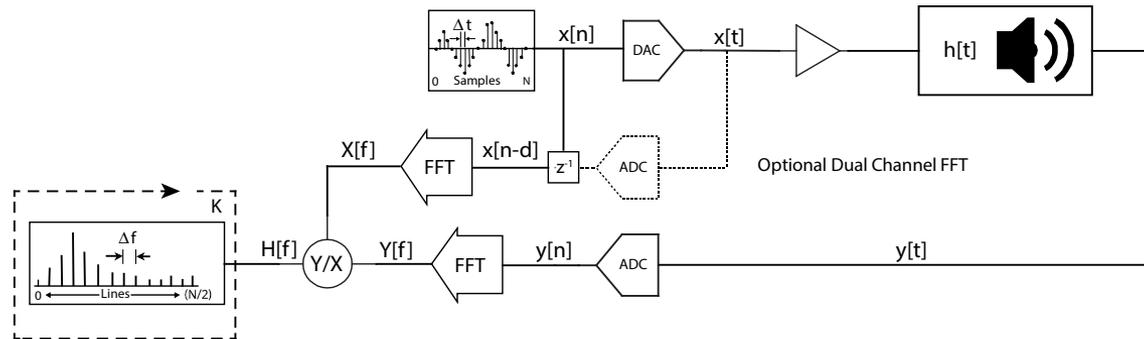
This might sound pretty straight forward, but these were groundbreaking developments around the turn of the century when they first started to appear in literature from Einstein and Wiener. [5]

Methods for Measurement

Transfer Functions – Direct Form



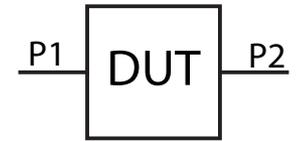
Two Port-Vector Measurement



- Gardner [5] contains many more DFT based implementation details on more complex statistical methods such as Bartlett-Welch, Wiener-Daniell, Blackman-Tukey, Channelizer, and Min Leakage. These are the methods which will be used in practical systems.
- As audio DSPs continue to offer more advanced filtering options, we typically also want to make full Transfer Function estimates, including the phase.
- This field of study is typically called System Identification and has incredibly diverse applications, which can make finding good generic sources of examples difficult, since they tend to be specialized.
- Although that basic equations and the diagram above seem to indicate it IS as simple as dividing the two spectrums Y/X . This very difficult in practice, as Y and X may not be a good stable estimates at any given moment.
- We can extend this implementation a little bit and do some rearranging, and arrive at how most commercial Dual-Channel FFT analyzers actually work.

Methods for Measurement

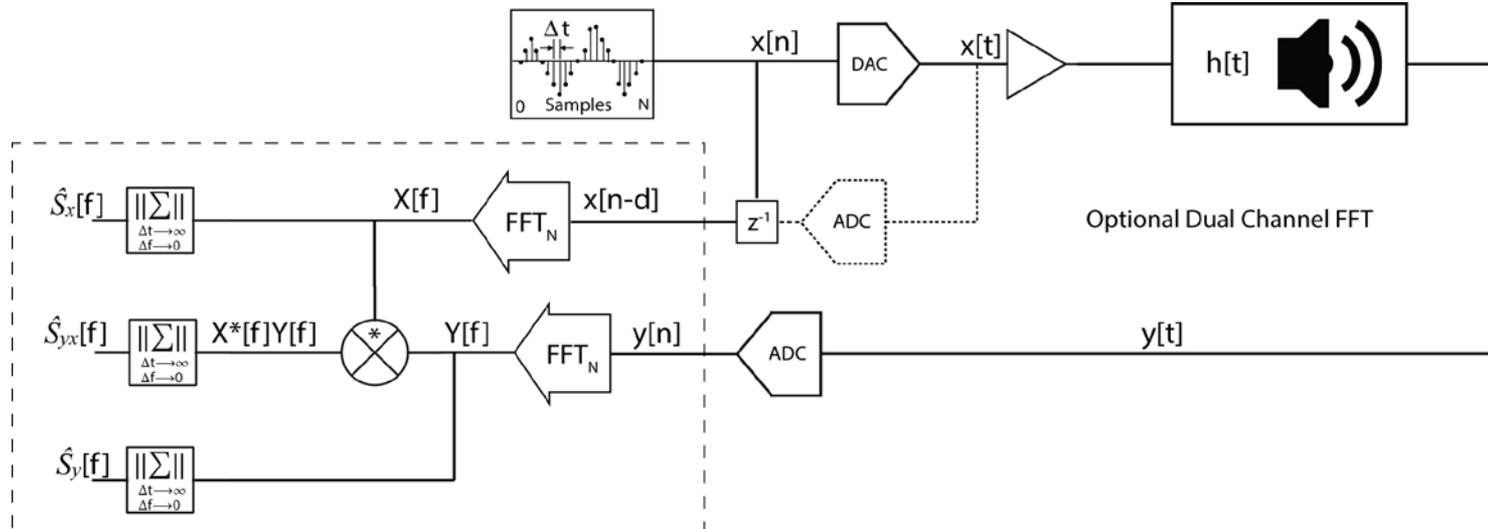
Transfer Functions – Direct Form



Two Port-Vector Measurement

$$\hat{H}[f] = \frac{\hat{S}_{yx}[f]}{\hat{S}_x[f]}$$

$$\hat{C}[f] = \frac{\hat{S}_{yx}[f]}{\sqrt{\hat{S}_x[f]\hat{S}_y[f]}}$$

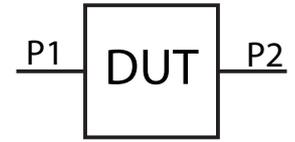


- Everything inside the dotted box is really implementation specific and the order and exact connections might be rearranged, depending on the statistical method used.
- Sigma blocks represent a statistical spectral method which converges to the limit spectrum such as those listed on the previous page from [5].
- We can imagine the FFT block feeds frames of N samples into the multiplier and statistical blocks, which do some form of averaging.

- $\hat{H}[f]$ is the transfer function estimate
- $\hat{C}[f]$ is the coherence function. A measure related to the error between $H[f]$ and $\hat{H}[f]$
- The middle leg is the Fast cross correlation of X and Y, there are some padding issues not addressed here. This could also be the FHT done in the time domain[4].
- I have to confess I've never personally implemented an analyzer like this, so if you see any errors let me know!

Methods for Measurement

Transfer Functions – Direct Form



Two Port-Vector Measurement

So, What is the Direct Method?

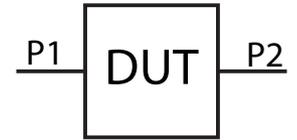
- Really, I am lumping a lot of different techniques together into the one big category.
- In a general sense, most stepped sine, MLS, and modern dual-channel FFT analyzers fit into this category.
- There is a huge amount of continuing research and improvements going on, especially with connecting the model directly to the measurement system.
- Each specific implementation has its own set of advantages and draw backs.
- If you really want to know the transfer function, this is the only way to setup all the proper equipment and measure it, that I am aware of. This would mean selection of all the spectral analysis parameters, stepped sine stimulus, in a anechoic chamber.

General Drawbacks to the Direct Method

- Distortion is distributed throughout the measured transfer function. This is a big one we'll talk about more with matched filtering.
- It takes some time for the statistical methods to converge.
- Typical PRN stimulus signals will correlate with interference noise.
- The longer the test must run to converge, the more chance there is of picking up correlated interference.
- The estimated phase SNR tends to be worse than deterministic stimulus signals used with matched filtering.
- Time variance and jitter in the measurement severely degrade the HF SNR in older MLS style implementations.

Methods for Measurement

Transfer Functions – Direct Form



Two Port-Vector Measurement

Questions / Additional Items / References

- [1] S. Stearns and R. David, “Signal Processing Algorithms in Fortran and C,” Prentice Hall, Englewood Cliffs, NJ 1993.
- [2] R. Bortoni, S. Filho, and R. Seara, “Comparative Analysis of Moving-Coil Loudspeakers Driven by Voltage and Current Sources,” 115th AES Convention, New York, NY, Oct 2003.
- [3] N. Iversen and A. Knott, “Small Signal Loudspeaker Impedance Emulator,” Journal of the Audio Engineering Society, Engineering Report, Vol. 62, No. 10, October 2014.
- [4] S. Müller and P. Massarani, “Transfer-Function Measurement with Sweep,” Journal of the Audio Engineering Society, Vol 49, No 6, June 2001.
- [5] W. Gardner, “Statistical Spectral Analysis: A Nonprobabilistic Theory,” Prentice Hall, Englewood Cliffs, NJ, 1988.

Chapter 2

Methods for Audio Measurements

One Port

- SPL Meter
- Spectrum Analyzer / RTA

Two Port – Transfer Function

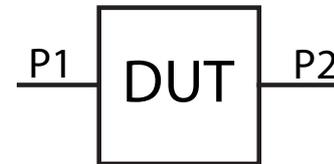
- Direct Form
- Matched Filtering

Device Under Test (DUT)



One Port-Scalar Measurement

The DUT does something and we measure it.

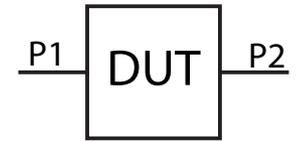


Two Port-Vector Measurement

We input something to the DUT on P1 and measure what the DUT does to it on P2

Methods for Measurement

Transfer Functions – Matched Filtering

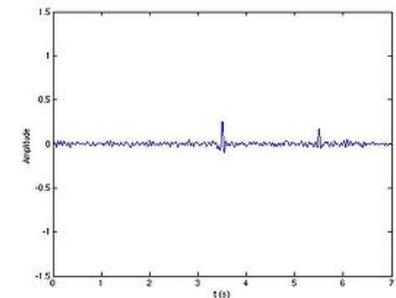
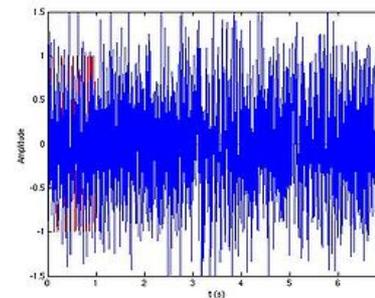
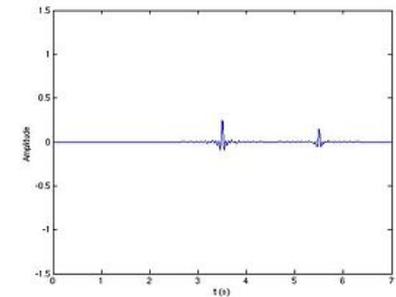
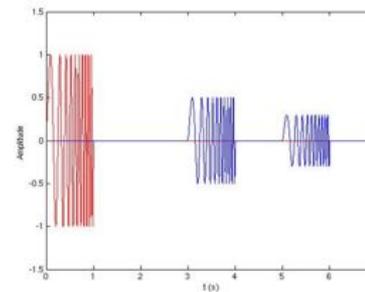


Two Port-Vector Measurement

A bit of History –

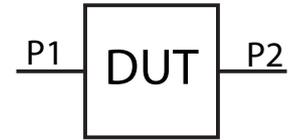
Matched Filtering and pulse compression, although I'm unclear on the original sources of its development, has its roots in Radar and Sonar Doppler signal processing.

- Pulse compression was developed for Radar and Sonar as an improvement to the tradeoff of range resolution and peak power output in traditional Doppler processing [7].
- In the audio industry, the first references appear with Heyser's TDS method and research around digital implementations [5], and later work by Farina [2],[6], and continued work in [3],[4].
- Heyser used hardware pulse compression to directly measure an approximation of a systems analytic impulse response [5].
- Farina proposed a digital matched filter for simultaneous measurement of impulse response and distortion [2].



Methods for Measurement

Transfer Functions – Matched Filtering



Two Port-Vector Measurement

How it Works-

A modern digital implementation of a matched filtering based transfer function measurement system is fairly straight forward.

First lets look at the signals and stimulus

Loose Definition of a Mono-Component Signal

$$s[t] = A[t]e^{j\phi[t]}$$

or

$$s[t] = A[t]\sin(\phi[t])$$

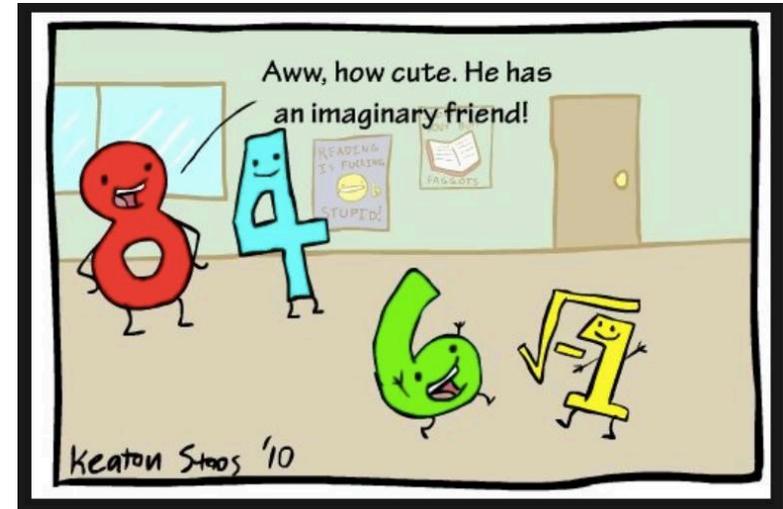
where $A[t]$ is the envelope and $\phi[t]$ is the phase

For signal synthesis purposes, it is more interesting to look at the Instantaneous Frequency

$$\omega[t] = \phi'[t]$$

and define our signal as

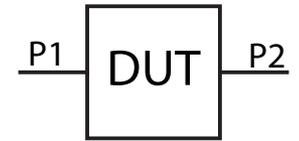
$$s[t] = A[t]\cos(\int \omega[t])$$



Since in audio systems we must deliver much more energy into the LF than the HF to achieve the same SNR, Farina and others proposed that the instantaneous frequency of our stimulus sweep increase exponentially.

Methods for Measurement

Transfer Functions – Matched Filtering



Two Port-Vector Measurement

- With the IF being exponential, the sweep will spend most of it's time gliding through the LF, hence more power will be delivered to the LF, improving the SNR.
- For reasons which will become clear later, when we talk about distortion, it is important that our signal have zero crossings at integer multiples of the start frequency.
- This can be accomplished by computing the sweep rate as follows [4], for an approximate length \hat{T} .

Let R be an integer such that

$$R = \frac{\hat{T}f_1}{\ln(f_2 / f_1)}$$

then L, the sweep rate, is

$$L = \frac{1}{f_1} R$$

The real length is then

$$T = \frac{1}{f_1} R \ln(f_2 / f_1)$$

The Instantaneous Frequency is

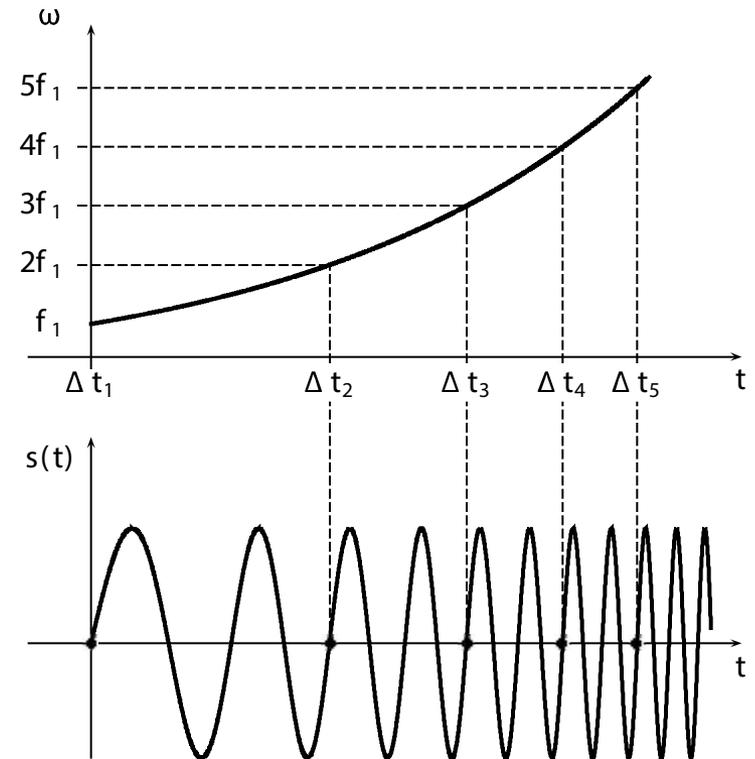
$$w[t] = f_1 e^{t/L}$$

The phase is

$$\phi[t] = \int w[t] = 2\pi f_1 (e^{t/L} - 1)$$

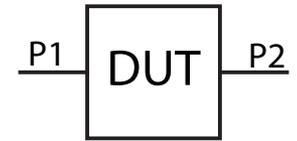
And our stimulus signal is then

$$s[t] = \sin(2\pi f_1 L (e^{t/L} - 1)) \quad \forall t \in [0, T]$$



Methods for Measurement

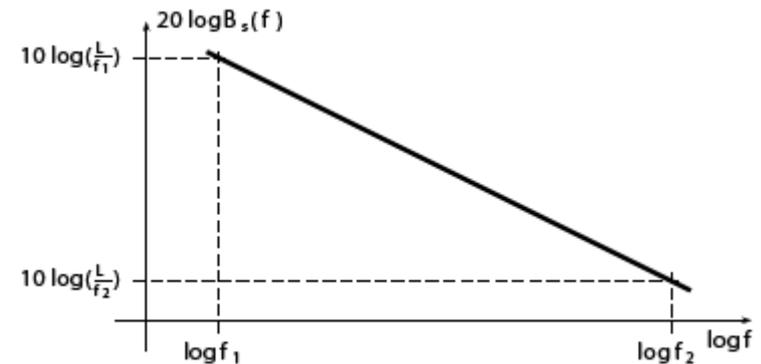
Transfer Functions – Matched Filtering



Two Port-Vector Measurement

- As you can see, we could really construct any IF curve we wanted and design specially weighted Matched Filter stimulus. For example, we could let $w[t]$ be a cubic spline that the user draws. We just need to be able to integrate it over T .
- Neglecting the bandlimited nature of the stimulus, the PSD will be like that of pink noise[4]. In reality, there will be edge effect due to finite length.
- Now that we have decided on our stimulus, we must construct its inverse.
- The referenced audio related papers indicate that we should be able to recover a perfect impulse from matched filtering, but that is impossible, and the error has continued to propagate in AES circles. This is really just a crazy mathematical reasoning error on their part. And could be considered a down side of transfer function measurement this way.
- I have never worked out the solution for a Exponential sweep, but the author of [7] worked out the solution for a linear sweep of length T , rate K , center f , and a delay of d .
- I guess it is a good idea to remember, just because it is published doesn't necessarily mean it is true. The solution in [7] matches very well with real measurements.

Asymptotic Spectrum of $s[t]$



Let $v[t]$ be our inverse Matched Filter
Some authors claim the following to be true
 $s[t] \otimes v[t] = \delta[t]$

where δ is the *Kronecker delta*

The first term is a all-pass phase warping

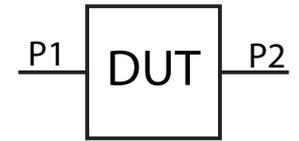
The second term, is a impulse like sinc function

$$H_{ap}[t] = e^{j2\pi(f(t-d) + \frac{K}{2}(t-d)^2)}$$

$$h[t] = H_{ap}[t](T \text{sinc}[KT(t-d)])$$

Methods for Measurement

Transfer Functions – Matched Filtering



Two Port-Vector Measurement

- Okay, back to constructing the inverse filter. The author in [4] had a great explanation which I will do my best to reproduce here.
- For sake of representation and connection with physical models, we will often use the Analytic Signal. Unlike the Fourier transform of a real signal, the transform of its analytic continuation does not have any negative frequency content.
- Analytic Continuation, in simple terms means we make stuff up that isn't there, but it needs to make sense. The Analytic continuation proposed by Cohen in [8] is simply:

$$z[t] = s[t] + jH\{s[t]\}$$

where $H\{\}$ is the Hilbert Transform

- If we do this, we now have a Complex signal which has no negative frequencies. This will come in real handy again when looking at Time-Frequency Representations of a digital signal. In this case we are using the analytic signal so we can draw conclusions from the instantaneous frequency and envelope functions.

$$F\{z[t]\} = Z[f] = B[f]e^{j\psi[f]}$$

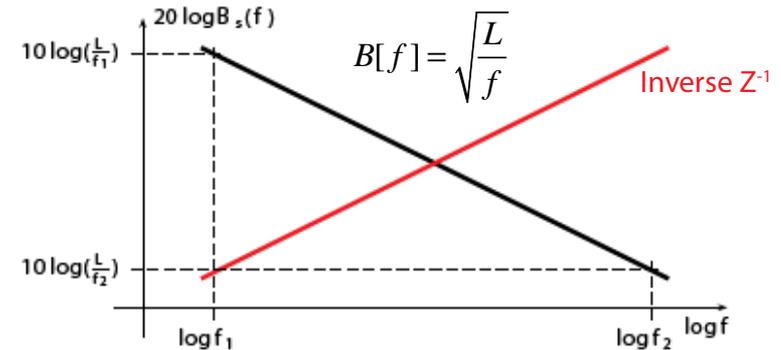
the inversion of $Z[f]$ is then

$$Z^{-1}[f] = \frac{1}{Z[f]} = \frac{1}{B[f]}e^{-j\psi[f]}$$

we want to find

$$z^{-1}[t] = a[t]e^{j\phi[t]}$$

Asymptotic Spectrum of $s[t]$



- Leaving out a lot of identities and algebra from Cohen [8], we end up with:

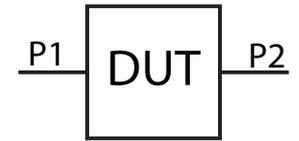
$$z^{-1}[t] = \frac{f_1}{L} e^{-t/L} e^{j\phi[-t]}$$

which implies

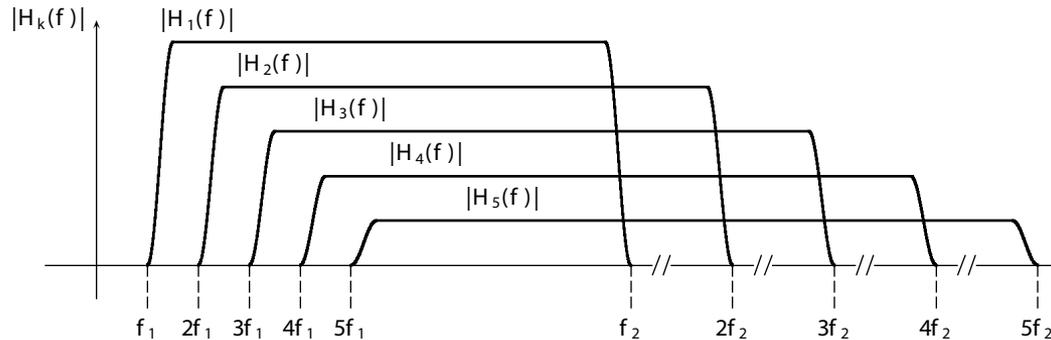
$$v[t] = \frac{f_1}{L} e^{-\frac{t}{L}} s[-t]$$

Methods for Measurement

Transfer Functions – Matched Filtering



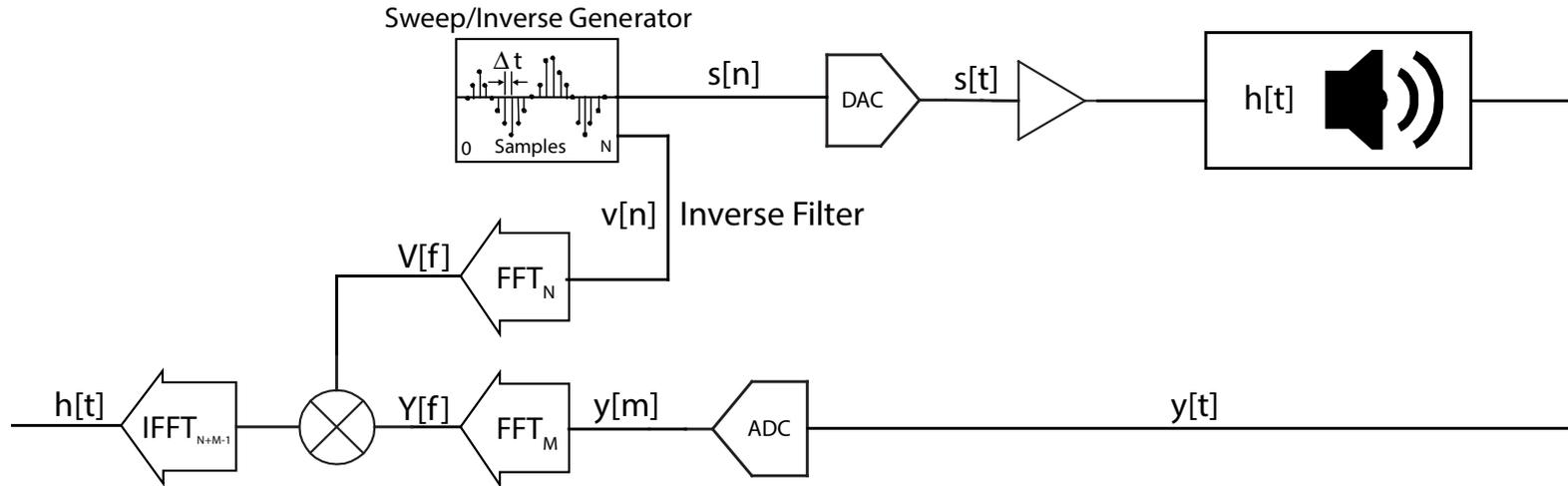
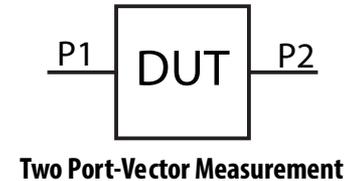
Two Port-Vector Measurement



- The author in [4] made another discovery which, as far as I can tell was original, that the frequency domain support of the inverse, should be much larger than that of the stimulus.
- Okay lets put the two together!
- If the idea is that we would like to resolve distortion products, since they will be possibly at frequencies greater than f_2 of our original sweep, we should create an inverse filter with support extending to the highest distortion product we would like to resolve.

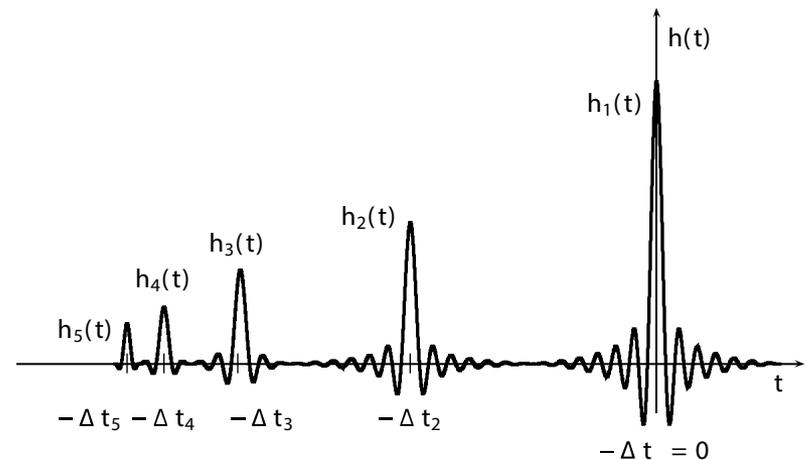
Methods for Measurement

Transfer Functions – Matched Filtering



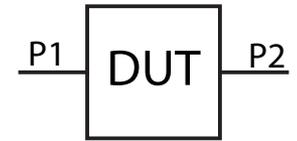
- The output is the system impulse response, depending on the implementation, it should be centered up with plenty of negative time before the impulse and plenty of positive time afterwards.
- Because of the phase of the sweep, certain types of distortion products should be spaced out in negative time from the fundamental impulse response.

$$\Delta t_m = L \ln(m)$$



Methods for Measurement

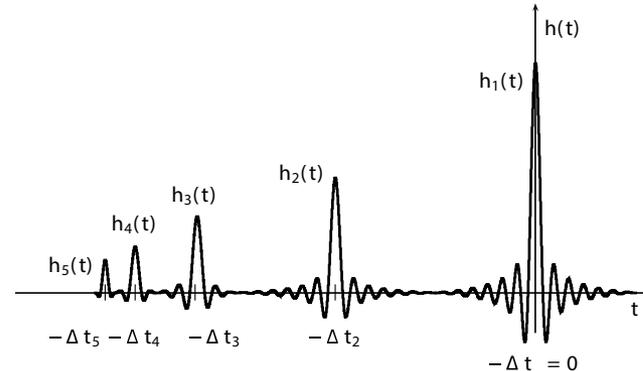
Transfer Functions – Matched Filtering



Two Port-Vector Measurement

Advantages of Matched Filtering Method

- You can get a good idea of the THD simultaneously with your Transfer Function
- Very Fast, No averaging
- Easy to implement highly accurate system on a variety of hardware platforms.
- Very good phase SNR over entire bandwidth
- Stimulus does not correlate well with most common noise sources. You can measure systems right through music playback and still have okay SNR.
- Although there is not a lot of active development, I think there is still a lot to discover here.

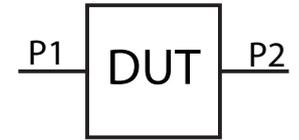


Problems with Matched Filtering Method

- If the system you are trying to measure, does not have well defined, finite support, you may not be measuring what you think you are.
- Since the impulse response recovered from matched filtering is convolved with “other stuff”, if the DUT is not band limited, my might be measuring the measurement system and not the DUT.
- You never get the real, high confidence, transfer function, sorry.
- Impulse Noise interferes with the measurement.
- No way to setup like your typical Dual-Channel FFT.
- No statistical averaging [6], only long sweeps.

Methods for Measurement

Transfer Functions – Matched Filtering



Two Port-Vector Measurement

Questions / Additional Items / References

- [1] S. Müller and P. Massarani, “Transfer-Function Measurement with Sweep,” Journal of the Audio Engineering Society, Vol 49, No 6, June 2001.
- [2] A. Farina, “Simultaneous Measurement of Impulse Response and Distortion with Swept-Sine Technique,” Audio Engineering Society, 108th Convention, Paris, France, Feb 19-22, 2000.
- [3] P. Dietrich, B. Masiero, and M. Vorlander, “On the Optimization of the Multiple Exponential Sweep Method,” Journal of the Audio Engineering Society, Vol. 61, No 3, March 2013.
- [4] A. Novak, “Identification of Nonlinear Systems In Acoustics,” Doctoral Thesis University of Maine, Le Mans, France, April, 2009.
- [5] M. Poletti, “Linear Swept Frequency Measurements, Time-Delay Spectrometry, and the Wigner Distribution,” Journal of the Audio Engineering Society, Vol. 36, No. 6, June, 1988.
- [6] A. Farina, “Advancements in Impulse Response Measurements by Sine Sweeps,” Audio Engineering Society, 122nd Convention, Vienna, Austria, May 5-8, 2007.
- [7] C. Ozdemir, “Inverse Synthetic Aperture Radar Imaging with MATLAB Algorithms,” Wiley, Hoboken, NJ, 2012.
- [8] L. Cohen, “Time-Frequency Analysis,” Prentice Hall, Upper Saddle River, NJ 1995.

Chapter 2

Methods for Audio Measurements

One Port

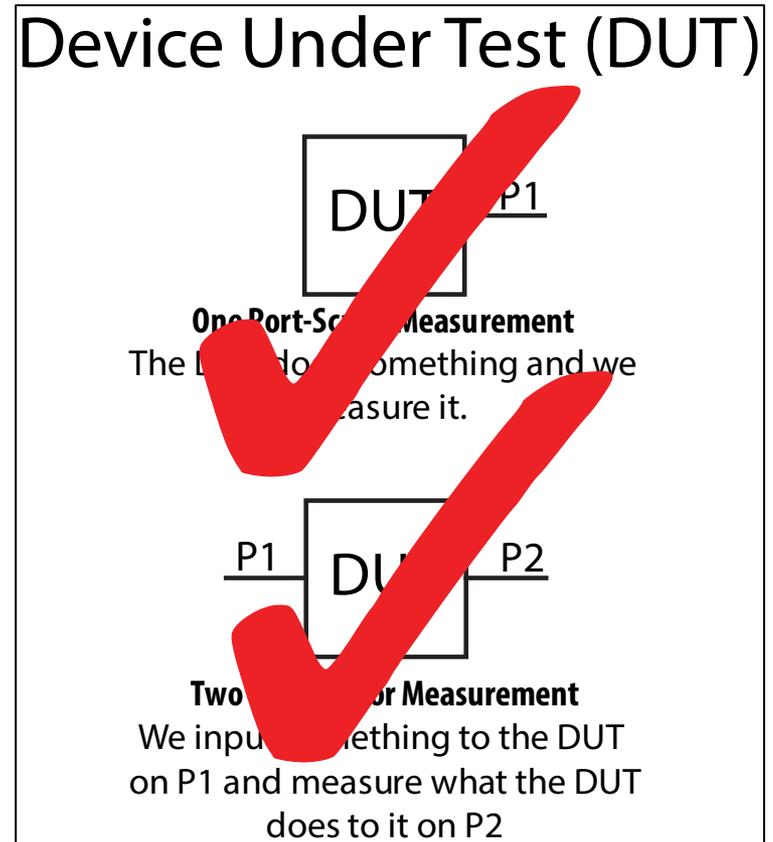
- SPL Meter
- Spectrum Analyzer / RTA

Two Port – Transfer Function

- Direct Form
- Matched Filtering

WHEW!

Now for some of the fun stuff.



Some Final Thoughts

- Distortion (THD)
 - Synchronous Detection
 - Matched Filtering
 - Rub and Buzz

- Impedance and Power



Distortion

- When working with LTI systems models, we tend to label any non-linearity or time variance as distortion.
- Like noise, we have modeled different distortion mechanisms and some standard tests to deal with them.
- Typical examples, THD, Intermodulation.
- Other mechanisms like, a rubbing voice coil, will generate non-harmonic distortion products, and in fact will pass right through any type of THD testing.

Demos:

- We will look at the compression driver on the PWT.
- THD and IMD in Signal Express
- Rubbing in Signal Express
- Matched Filter responses

Impedance and Power

- I was not able to get this demo put together in time. Really sorry.
- Many companies measure Impedance of drivers much the same as any other transfer function measurement.
- Devices like the LinearX VI box provide a 4-wire resistor to measure current.
- Any multi-channel setup and then measure the voltage and current transfer function.
- Be careful using such devices, as high common mode voltages can be present on your DAQ device and could damage it.
- The demo I wanted to bring with was a more advanced custom device, an integrated PWM amplifier, coherent attenuated out, Hall effect current probes, and a precision was to measure high voltages, which could guarantee phase linearity over the entire Audio band.
- I am really interested in exploring this area further
- Impedance measurements are really integral to the development of any loudspeaker product.
- We typically use the Impedance analyzer more than any other tool while developing loudspeakers, enclosures, cross-overs.
- Many times it is much easier to identify defects and nonlinearities in loudspeakers in the impedance domain, while they may remain masked in the SPL measurements.
- I am really interested in further exploring motor development through impedance analysis. We have developed a number of drivers where Impedance data was integrated with FEM tools to further optimize designs.
- Currently we use two different versions of the same custom hardware design, pretty close in functionality to the VI box. One version is part of a multichannel isolated high current, high voltage setup for measuring long term power vs. xyz.

Impedance and Power

- The key to determining power for actual measurements and not AES specifications, is your measurement program design.
- You need to be able to get reading of instantaneous voltage and current simultaneously , as well as the RMS quantities.
- From those measurements it is relatively easy to determine all the complex power quantities.
- If your measurement system can also correlate that with Temperature, SPL, and etc., it makes the instrument much more useful.

Questions or Comments?

- Thanks to everyone for coming